

# An intertemporal analysis of foreign borrowing for developing economies\*

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# An intertemporal analysis of foreign borrowing for developing economies

## *Abstract*

Foreign capital flows have become increasingly important in financing investment and growth in developing countries. Foreign capital flows can, however, be volatile as is evident from recent financial crises. This paper analyzes the borrowing behavior of a developing economy that relies on foreign borrowing for its capital formation. In particular, this paper investigates the implications of different lending policies of international financial institutions. When the borrowing interest rate increases with the absolute level of foreign debt, there is high possibility for developing countries to be on the growth path toward the zero consumption. When the borrowing rate depends on foreign-capital/total-capital ratio, however, the economy always moves toward the stationary state irrespective of the level of initial per capita capital, washing away the possibility of the path toward zero consumption. The results imply that the lending policies of the financial institutions in industrial countries need to be modified in such a way that the lending rates increase gradually with the level of foreign debt relative to the size of the borrowing country.

**Keywords:** Foreign borrowing, Imports of investment good, financial crisis

**JEL Classification:** C61; F34; F43

## 1. Introduction

Foreign capital flows have become increasingly important in financing investment and growth in developing countries. The foreign debt problem, however, has continued to pose a threat to the integration and fabric of the international financial system, as foreign capital flows can be volatile as is evident from recent financial crises. At the core of the Asian crisis, for example, were large-scale foreign capital inflows into Asian financial systems that became vulnerable to panic. (Radelet and Sachs, 1998). Questions arise again as to whether those countries have been borrowing too much and whether creditors have overextended themselves. In this context, it is more important than ever to gain insights into the role of lending policies of the financial institutions in industrial countries that may affect the trends and direction of such flows.

Traditionally, foreign borrowing is seen as a source of increased resources for investment in the growth-cum-debt framework. A well-known example is the Harrod-Domar model. However, this approach is not designed to tackle the issue of how much a country should borrow, in that foreign borrowing is modeled to fill the gap between the required level of investment and the level of domestic savings, and also to service outstanding debt. Bardhan (1967), Hamada (1969) and Hanson (1974) are early studies that analyze the optimal level of foreign borrowing by basing the models on the context of intertemporal optimizing principles. Yet, McDonald (1982) observes that the emphasis on growth by investment tends to neglect the role of foreign borrowing in achieving a more efficient intertemporal allocation of consumption. In fact, the recent literature has paid more attention to the use of foreign borrowing for consumption purposes. In a standard setup, foreign borrowing is used mainly to smooth the consumption path over time where consumption and investment decisions are made independently of each other under the assumption of a small open economy facing a given world interest rate.<sup>1</sup>

The purpose of this paper is to recast the investment role of foreign borrowing in a

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<sup>1</sup> See Obstfeld and Rogoff (1996: 19) for rationales underpinning the separation of consumption from investment.

modern intertemporal framework. In particular, this paper investigates the implications of different lending policies of international financial institutions. Along this line of inquiry, the present paper develops a dynamic optimizing model of consumption and foreign borrowing for a developing economy that relies on foreign borrowing for its capital formation. The assumption of a constant interest rate restricts the time discount rate to be equal to the interest rate, a rather unrealistic assumption. Several variations have been proposed in the infinite-horizon framework in order to alleviate this constraint. Sen and Turnovsky (1989) introduce adjustment costs of investment to avoid this constraint. Obstfeld (1982) allows a variable rate of time preference by endogenizing it (making a function of the utility level) through the introduction of Uzawa (1968) preferences. Alternatively, the overlapping generations model allows for effects from finite horizons by assuming that people live a fixed number of discrete periods. Blanchard (1985) retains the essence of the finite-horizon idea in a more tractable framework by assuming a finite probability of death. In this stream, the world interest rate and the rate of time preference do not necessarily have to be equal, and the difference between them becomes a function of financial wealth. Barro and Sala-i-Martin (1995: Chapter 3) provides a comprehensive review on these extensions.

A distinctive feature of this paper is to consider the effect of country risk on the interest rate faced by small borrowing economies on world capital markets. Bardhan (1967) was one of the first economists to introduce the assumption that small borrowing countries face a world interest rate that increases with the level of foreign debt. As foreign indebtedness grows, so does the risk of default; to compensate, lenders charge a premium that raises the marginal cost of borrowing over the safe lending rate. The supply curve of funds on world capital markets is therefore upward sloping. Bardhan's approach has been adopted in numerous studies including papers by Obstfeld (1982), Edwards (1984), Sachs (1984), Bhandari *et al.* (1990), Senhadji (1997), Fisher (1995), and Agénor (1997).

We first incorporate this idea by assuming that the rate of interest faced by the debtor country rises with the level of foreign debt. The effect of such a constraint on borrowing changes the dynamics in a fundamental way. In particular, the marginal cost of capital facing

the agent, and therefore determining his/her investment decisions, is now dependent on the outstanding stock of debt. The results show that the economy can reach the stationary state when the initial per capita capital is large enough. However, the economy fails to reach this stationary state when the initial per capita capital is low, even if such a stationary state exists, and instead reaches another stationary state with zero consumption. The former stationary state is realized if the productivity is high or the time discount rate is low. When the productivity is low or the time discount rate is high, then the economy reaches the latter stationary state even if the import of foreign investment good is positive.

Next, we introduce an alternative assumption that the interest rate depends not on the absolute level of foreign capital but on foreign-capital/total-capital ratio as done by Chatterjee and Turnovsky(2004, 2005). In this case, the economy always moves toward the stationary state irrespective of the level of initial per capita capital, washing away the possibility of the growth path toward the zero consumption. Hence, this assumption radically changes the previous results.

The results imply that in order to reduce the risk to emerging market countries, the lending policies of the financial institutions in industrial countries need to be modified in such a way that the lending rates increase gradually with the level of foreign debt relative to the size of the borrowing country. This is simple but is not well practiced in reality because of imperfect information (the lack of consistent and reliable macroeconomic data) and/or moral hazard (the encouraging of imprudent or unsustainable behavior by the creditors with the perception that international community will rescue risky investments in the case of financial crisis).

The remainder of this paper is organized as follows. In Section 2, we develop a basic framework of our model, and in Section 3, by introducing an assumption that the borrowing rate increases with the outstanding level of foreign capital in a developing country, we show the existence of the two kinds of stationary states. In Section 4, we introduce an alternative assumption that the interest rate depends on foreign-capital/total-capital ratio, and show the existence of unique stationary state. Section 5 concludes the paper and provides some useful

implications.

## 2. Basic Framework of Model

We consider the standard borrowing problem facing a planning authority of a small open economy, where it is expected to maximize the discounted utility streams of per capita consumption. The economy produces unique homogeneous good which can be either consumed or accumulated as capital stock. This implies that there exists no international trade in good. The country can borrow to import additional capital stock which is identical with the domestic one. It is assumed that foreign borrowing is exclusively utilized to augment domestic capital stock with a variable interest rate which increases with the increase in the foreign capital stock. In short, the home country can import capital  $k_f$  from the foreign country if necessary.

Production of the homogeneous good occurs using capital and labor according to a neoclassical production function exhibiting constant returns to scale. Per capita output is:

$$q_t = f(k), \quad (1)$$

where  $k$  is the capital–labor ratio in the production of the composite good. The production function satisfies the following usual assumptions:

$$\begin{aligned} f'(k) &> 0, \quad f''(k) < 0, \\ f(0) &= 0, \quad f(\infty) = \infty, \\ f'(0) &= \infty, \quad f'(\infty) = 0. \end{aligned}$$

Resources are fully employed and the labor is fixed at  $L = L_0$ . Of the total amount of capital used in the country, part is domestically owned capital and the rest is foreign capital:

$$k = k_d + k_f, \quad (2)$$

where  $k_d$  and  $k_f$  are the domestic capital–labor ratio (domestic capital per worker) and the foreign capital–labor ratio (foreign capital per worker), respectively.

Per capita income in period  $t$ ,  $y_t$ , is given by  $q_t$  net of per capita interest payments on

outstanding foreign debt in the corresponding period:

$$y_t = q_t - rk_f, \quad (3)$$

where  $r$  denotes the real rate of interest on foreign debt. Per capita domestic investment in period  $t$ ,  $i_d$ , is the difference between per capita income and per capita consumption:

$$y_t - c_t = i_d. \quad (4)$$

Assuming no capital depreciation, the net change in the level of domestic capital–labor ratio in period  $t$  is new domestic investment per person:

$$\dot{k}_d = i_d = f - rk_f - c. \quad (5)$$

The felicity function is specified to be

$$u(c) = \frac{1}{1-\sigma} c^{1-\sigma}, \text{ where } 0 < \sigma < 1. \quad (6)$$

### 3. The Case of $r = r(k_f)$ with $r'(k_f) > 0$

Here we assume that the interest rate in the home country rises with the levels of foreign capital per worker:

$$r = r(k_f), r' > 0. \quad (7)$$

Assumed further is that the interest rate changes at an increasing rate so that

$$r''(k_f) > 0. \quad (8)$$

(1) First we consider the case where the representative consumer takes interest rate  $r$  as given even if it changes according to (7), since as a microeconomic agent he/she thinks interest rate to be determined by markets (macroeconomic environment).

The planning problem faced by a representative agent with an infinite terminal time and positive discount rate is to choose paths for consumption ( $c$ ) and foreign capital stock ( $k_f$ ) as control variables and ( $k_d$ ) as state variable so as to

$$\text{Max}_{c, k_f} \int_0^{\infty} e^{-\rho t} u(c) dt \quad (9)$$

subject to

$$\dot{k}_d = f(k) - c - rk_f. \quad (5)$$

This is solved by taking the current value Hamiltonian

$$H = u(c) + \lambda(f(k) - c - r(k_f) \cdot k_f)$$

and obtaining the first order conditions

$$c^{-\sigma} = \lambda, \quad (10)$$

$$\dot{\lambda} = (\rho - f'(k))\lambda, \quad (11)$$

$$f'(k) = r \quad (12)$$

and the transversality condition (TVC)

$$\lim_{t \rightarrow \infty} k_d \lambda e^{-\rho t} = 0. \quad (13)$$

From (12) we observe

$$k = k(k_d) +$$

with  $dk/dk_d = 1/(dk_d/dk) = 1/(1 - dk_f/dk) = r'/(r' - f'')$ , so that  $0 < dk/dk_d < 1$  and hence  $dk_f/dk_d < 0$  and  $g'(k) = r'(k_f)k_f'k_f + rk_f' < 0$  where  $g(k_f(k)) = r(k_f) \cdot k_f$  hold.

From (12), there exists  $k = k_M = k_d$  such that  $f'(k_M) = r(0)$ , i.e., the maximum value of  $k$  where foreign capital  $k_f$  becomes zero. In short, an increase in  $k_d$  decreases the foreign capital  $k_f$  but increases the total capital stock  $k(=k_d + k_f)$ .

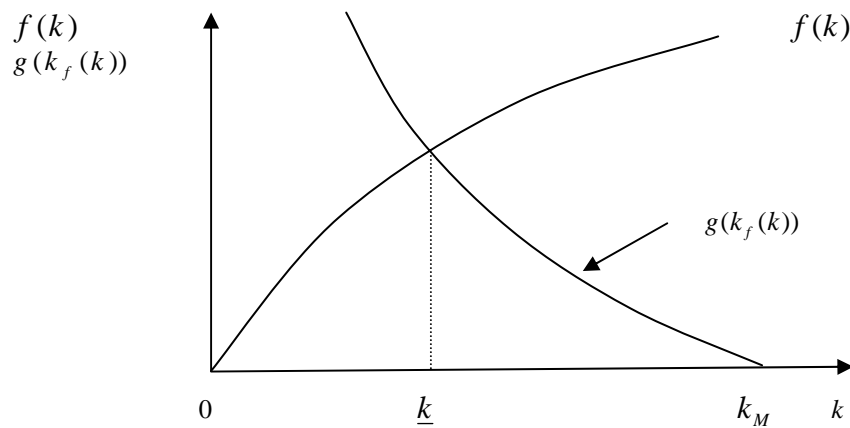
From (11) and (12), we obtain

$$\dot{c}/c = \sigma^{-1}(f'(k) - \rho). \quad (14)$$

Using (5) with  $\dot{k}_d = 0$ , the  $\dot{k}_d = 0$  curve is expressed as

$$c = f(k) - g(k_f(k)). \quad (15)$$

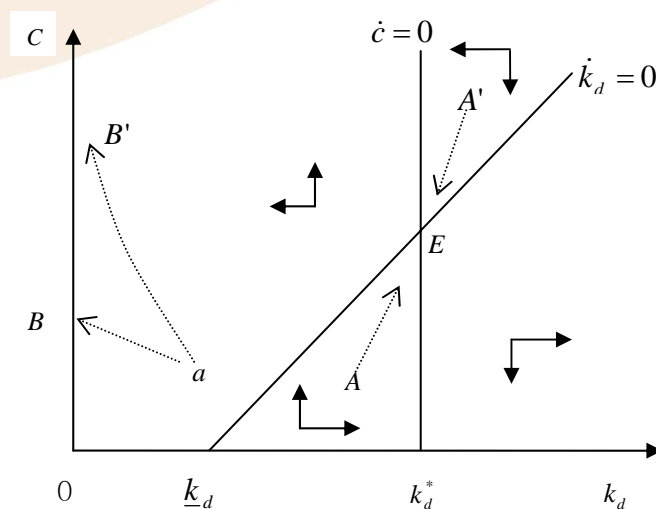




[Fig. 1]

Recalling  $g' > 0$  and  $dk_f/dk < 0$ , we obtain from (15) and Fig. 1 that there exists  $k = \underline{k}$  such that  $c = 0$  at  $k = \underline{k}$  and  $c > 0$  for  $k > \underline{k}$  with  $dc/dk > 0$  for  $k \geq \underline{k}$ . Recalling further that  $k$  depends on  $k_d$ , we obtain along the  $\dot{k}_d = 0$  curve,  $dc/dk_d > 0$  for  $k_d > \underline{k}_d$  where  $\underline{k} = k(\underline{k}_d)$ , i.e.  $\underline{k}_d$  is the value of  $k_d$  corresponding to  $k = \underline{k}$ , and the  $\dot{k}_d = 0$  curve is positively sloped as drawn in Fig. 2.

Next, we consider the  $\dot{c} = 0$  curve. This is obtained from (14), with  $\dot{c} = 0$ , as a vertical line with  $k_d = k_d^*$ , where  $f'(k^*) = \rho$  and  $k^* = k(k_d^*)$ , i.e.  $k_d^*$  is the value of  $k_d$  such that  $k^* = k(k_d^*)$  holds and at  $k = k^*$ ,  $f'(k^*) = \rho$  holds, as drawn in Fig. 2.



[Fig. 2]

First, we consider

Case 1.  $\underline{k}_d < k_d^*$ ,

That is, two curves  $\dot{k}_d = 0$  and  $\dot{c} = 0$  intersect at  $E(k_d^*, c^*)$ . Then we obtain the phase diagram as drawn in Fig. 2 from (14) and (15).

From Fig. 2, it is seen that

- (i) if initial per capita domestic capital  $k_{d0}$  is larger than  $\underline{k}_d$  but less than  $k_d^*$ , i.e.,  $\underline{k}_d < k_{d0} < k_d^*$ , then the economy is on the growth path with both  $k_d$  and  $c$  increasing toward the equilibrium  $E$ .
- (ii) if  $k_{d0}$  is larger than  $k_d^*$ , i.e.,  $k_{d0} > k_d^*$ , then again the economy is on the stable path with both  $k_d$  and  $c$  decreasing toward  $E$ ;
- (iii) if initial  $k_d$  is less than  $\underline{k}_d$ , then the economy reaches another stationary state where the consumption is zero. That is, as drawn in Fig. 2,  $k_d$  keeps decreasing and  $c$  and  $k_f$  keep increasing, but  $k_d$  becomes zero (i.e.,  $k = k_f$ ) in a finite time  $t = T < +\infty$ , and then  $c$  and  $k_f$  also become zero at  $t = T$ . That is, at  $t = T$ , all foreign capital stock  $k_f$  is withdrawn from the home country.

Here we note for (iii) the path  $(k_d, c) \rightarrow (0, \infty)$  as  $t \rightarrow +\infty$  (shown by arrowed dotted line  $aB'$ ) violates the transversality condition. One transversality condition for this case is that the present value Hamiltonian  $\tilde{H} = e^{-\rho t} H$  vanishes as  $t \rightarrow +\infty$ .<sup>2</sup> However  $\tilde{H}$  does not vanish if  $k_d \rightarrow 0$ ,  $c \rightarrow +\infty$ .<sup>3</sup>

We note that (iii), i.e., the economy moves toward the zero consumption if initial per capita domestic capital  $k_{d0}$  is smaller than  $k_d^*$  even if the stationary state  $E^*$  which

<sup>2</sup> See, e.g., Léonard and van Long (1992) Chapter 9.6.

<sup>3</sup> Let  $x = u(c)e^{-\rho t}$ , then  $\dot{x}/x = (u'/u)\dot{c} - \rho = (1 - \sigma)^{-1}\dot{c}/c - \rho = (1 - \sigma)^{-1}\sigma^{-1}(f' - \rho) - \rho \gg 0$  as  $k_d \rightarrow 0$ . (As seen from  $k = k(k_d)$ ,  $k$  decreases as  $k_d \rightarrow 0$ . Hence we may assume  $\dot{x}/x \gg 0$  as  $k_d \rightarrow 0$ .)























