

# Expert Advice, Control, and Heterogeneous Beliefs

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## Abstract

This paper studies the effects of overconfidence in an investment-decision setting. A risk-averse agent privately observes information relevant to an investment decision, that he can report to a principal. In a standard common-priors setting, the optimal contract provides full insurance to the agent: the principal pays a fixed wage to the agent, asks him to reveal his information, and implements the efficient investment rule. When the agent overestimates the expected revenue of the project following investment, however, he is willing to “wager” on success against the (relatively pessimistic) principal, and hence bear some project risk in equilibrium. In addition, because what the principal considers to be the optimal investment rule is too conservative according to the agent’s beliefs and the agent holds some stake in the choice of investment rule, he will accept a lower fixed payment in exchange for a more liberal investment rule. This can be interpreted as giving more control to the agent. It is somewhat counterintuitive that, the principal will surrender more control to an agent with whom she disagrees more sharply.

**Keywords:** overconfidence, heterogeneous beliefs, expert advice.

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# 1 Introduction

An important function of a firm's manager is to guide investment decisions; an experienced manager is likely to be better informed than the firm's owner regarding the profitability of potential investment projects. If the manager's attitudes towards risk differ from the owner's, however, full delegation of investment decisions may not be in the owner's best interest.

Consider, in particular, a case in which a risk-averse manager is hired by a risk-neutral owner. Imagine that the manager can share his private information regarding the project's expected profits with the owner. After receiving the manager's report, the owner then decides whether or not to invest in the project. In this principal-agent setting, under the assumption that manager and owner agree about the distribution of profits conditional on the manager's information, the manager's risk aversion does not pose an incentive problem. The owner can offer full insurance to the manager (i.e. offer him a fixed payment), and simply ask him to act with the owner's best interest in mind.

The common-priors assumption implicit in this logic is a strong one, however—in particular when the manager has some stake in the outcome of the investment decision following his recommendation. Studies in psychology suggest that individuals tend to be overly optimistic when evaluating their own skills and ability, and the likelihood of favorable outcomes. This bias seems to be stronger when evaluating outcomes that affect one's personal well-being. See Taylor and Brown (1988) for a discussion of the findings in psychology research regarding this bias. In our example, the manager's utility can be indirectly affected by the outcome of the investment decision (e.g. through ego, reputation). If the manager overestimates the returns to investment relative to the owner's beliefs, and his payment can be made contingent on the project's outcome following investment, then he will not receive full insurance in equilibrium.<sup>1</sup> In this case, the manager will in fact “wager” to some extent against the owner, so that he receives outcome-contingent pay in equilibrium. Because full insurance is no longer the equilibrium contract, the manager's well being is affected by the project's outcome following investment, and therefore by the owner's decision of whether or not to invest following the manager's recommendation. Disagreement about the expected returns to investment implies, in most instances, disagreement about the optimal course of action. As a result, the manager may be willing to “pay” for some degree of control over the investment decision. In contrast to the common-priors setting, in the presence of heterogeneous beliefs the investment rule implemented in equilibrium will almost surely not be the efficient investment rule according to the owner's beliefs. I briefly discuss the implications of this result in an insider/outsider (e.g. manager/consultant) setting in the concluding section.

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<sup>1</sup>Gervais, Heaton and Odean (2003), and Van den Steen (2005) also study the effects of overconfidence/optimism in investment-decision settings. Some of the results in this paper are consistent with their findings.

Section 2 introduces a simple model that allows us to explore the effects of agent overconfidence in an investment-decision setting. I follow the basic structure employed by Holmstrom and Ricart i Costa (1986). The most notable characteristic of the model is that the agent privately observes some signal that is correlated with the expected profits of a potential investment project and can share that information with his employer (the principal). Principals compete to hire the agent by making simultaneous contract offers, which results in zero expected profits for the principal in equilibrium. A contract offer specifies both the payment scheme and the investment rule that will be followed (whether or not investment is undertaken following a given report by the agent). The expected revenue of the project depends stochastically on both the signal, which is private information to the agent, and on the agent's ability, which is unknown to both parties. The agent can choose whether to report the observed signal to the principal, but cannot misrepresent information.<sup>2</sup> Then, the principal invests according to the agreed-upon investment rule. If the agent chooses not to report the signal, no investment will be made: the agent has "veto power" after observing the signal, but the principal does not.<sup>3</sup> Once outcomes are realized, payments are distributed according to the terms of the contract, and the agency relationship ends.

Section 3 isolates the issue of the equilibrium payment scheme from the issue of the equilibrium investment rule (which we incorporate in Section 4 for tractability). In a common-priors setting, the equilibrium payment scheme is very simple: the principal offers full insurance to the agent, so that the agent is paid his expected productivity, and the agent always reveals his private information in equilibrium. If, however, the agent overestimates his ability, and thus the expected revenue of the project following investment relative to the principal's beliefs, then the equilibrium payment scheme will be outcome-dependent. We allow principal and agent to hold heterogeneous beliefs, in particular, about the distribution of revenue conditional on each signal realization. In equilibrium, the agent receives a higher payment following those realizations of revenue that he deems more likely than the principal does, and in return accepts a lower payment following those realizations that he believes are relatively unlikely. Even though the agent is risk averse, some degree of "wagering" of this kind will always be observed in equilibrium. A corollary of this result is that a more-overconfident agent will tend to receive a higher payment in the event that no investment is

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<sup>2</sup>This is an important assumption in Holmstrom and Ricart i Costa (1986), that I maintain for tractability. The possibility of information misrepresentation is an interesting and relevant problem. Stocken (2000), for example, studies the credibility of information disclosure in a repeated cheap-talk game.

<sup>3</sup>Note that a risk-averse agent might prefer that no investment be made if his remuneration depends on project outcome. Allowing the agent to veto an investment project forces the principal to internalize this issue when designing the contract. If there are enough projects with large negative expected profits, it will be optimal for the principal to not invest following "no report" by the agent.

made, as long as it does not destroy the agent's incentives to fully reveal his private information. An overconfident agent overestimates his expected payment and his expected utility following the decision to invest; because he disagrees with the principal about his (unknown) ability but not about the ex-ante distribution of the signal (which both principal and agent know *ex-post* in equilibrium), he receives as much insurance as possible in terms of signal variability. Wishing to smooth his consumption across "invest" and "do not invest" events, a more overconfident agent will prefer a higher payment conditional on no investment, consistent with his expectation of higher payment if investment is undertaken.

Section 4 includes the investment rule as part of the optimization problem implicit in the equilibrium contract offer. Given that the agent receives full insurance in a common-priors setting, he is indifferent as to the implemented investment rule in that case. The principal thus implements the rule that she considers optimal, which, given her risk neutrality, is in fact the efficient rule. As a consequence of heterogeneous beliefs, the agent's payment is contingent on the outcome of investment whenever it is undertaken. Two sources of distortion (relative to the efficient rule according to the principal's beliefs) affect the equilibrium investment rule. First, the principal's objective function will almost surely no longer be aligned with efficiency. For the risk-averse agent to be willing to reveal his private information when the investment rule dictates investment (i.e. for him not to veto the project), it must be the case that he evaluates his *expected payment* conditional on investment as higher than what he would receive if no investment was made—which is a riskless payment. If the agent's expected payment conditional on investment is higher (or lower) according to the principal's beliefs as well, then the principal is relinquishing some of the marginal benefits of investment to the agent. As a consequence, the principal's objective function will, almost surely, not be aligned with efficiency. Second, if the agent believes that his *expected utility* will be higher conditional on investment than when no investment is made, he will be willing to forego some of his remuneration in exchange for a more liberal investment rule. The agent is "willing to pay for control" in that case, and the implemented investment rule will in fact be skewed away from what the principal would optimally choose if her decision was unilateral and non-contractible, and towards what the agent believes to be the optimal investment rule.

Section 5 provides an example of the equilibrium contract when the agent has an exponential utility function, and the productivity shocks are normally distributed. This purposefully simple example further illustrates the intuition behind the results of the model.

Section 6 concludes, and discusses a testable implication of the presence of overconfidence in an expert-advice setting, given that firms tend to rely on both outsiders' and insiders' advice to guide their investment decisions.

## 2 Model Setup

Consider a simple one-period expert-advice model, in which an agent gives information to a principal, who in turn includes the agent’s “expert advice” in her decision-making process. We will follow the notation and basic model structure used by Holmstrom and Ricart i Costa (1986), applied to a one-period setting and allowing for the agent’s payment to be contingent on same-period outcome.

The principal has the possibility of investing in a given project. At the beginning of the agency relationship, the agent observes a signal  $s$  that is correlated with the project’s profitability. The profitability of the project also depends on the agent’s ability. The project’s revenue  $y$ , conditional on a decision to undertake investment, can be written as

$$y = s + \varepsilon,$$

, where  $s$  is privately observed by the agent and has cumulative distribution function  $N$ , and the distribution of  $\varepsilon$  depends on the agent’s ability—which is unknown to both parties. Under the assumption of common prior beliefs (which we will relax below), principal and agent share the belief that  $\varepsilon$  has cumulative distribution function  $F$  with mean  $\mu$ .<sup>4</sup> The agent can decide whether or not to report the true value of  $s$  (which can be interpreted as whether or not to “recommend” investment), but cannot misrepresent information in the report. The principal can always guarantee zero revenue by not investing. The principal is risk neutral, and his objective is thus to maximize expected profits:

$$\max \mathbb{E}[y - w]$$

where  $w$  stands for the agent’s “wage”—a money transfer to the agent. The agent is risk averse, and his objective is to maximize his expected utility of consumption:  $\max \mathbb{E}[u(c)]$ . Assume that the agent’s only income is his wage, so that the agent’s objective can be expressed as

$$\max \mathbb{E}[u(w)].$$

Assume that several principals have access to this type of investment projects, and compete to contract with the agent.<sup>5</sup> The timing of the model is as follows: first, principals make simultaneous

<sup>4</sup>Holmstrom and Ricart i Costa (1986) assume that the agent can be either talented or not, and the prior probability of the agent being talented is  $p$ . In their model,  $\varepsilon \sim G$  with mean  $\mu_G$  if the agent is talented ( $G$  for “good”), and  $\varepsilon \sim B$  with mean  $\mu_B$  if the agent is not talented, where  $\mu_G > \mu_B$ . This specific form is one way to parametrize agent ability, particularly useful in a career-concerns setting; the generalization that  $\varepsilon \sim F$  with mean  $\mu$  allows for a more intuitive presentation of the results of this paper.

<sup>5</sup>The results in a setting where one principal makes a take-it-or-leave-it offer to an agent are nearly identical. See de la Rosa (2006a) for an extensive explanation. In short, these are dual problems, with the main difference being who retains the expected gains from trade.

contract offers to the agent. A contract offer consists of an investment rule and a payment schedule to the agent. The agent then chooses which offer (if any) to accept. When and if the agent chooses to accept a contract offer, he privately observes signal  $s$ , and chooses whether or not to report it to the principal. If the agent reports the signal, investment is undertaken or not according to the agreed-upon investment rule. If investment is undertaken,  $\varepsilon$  is then realized and correctly inferred by both parties (given that project profitability is publicly observed). There is no investment if the agent chooses not to report  $s$ . Payoffs are then distributed according to the agreed-upon payment schedule, and the agency relationship ends. If the agent chooses not to accept any contract, no project will be undertaken, and the players receive payoffs according to some outside option. I assume that the agent's outside option is low enough so that he always accepts some contract offer in equilibrium.

The equilibrium contract solves

$$\begin{aligned} \max \mathbb{E}[u(w)] \\ \text{s.t. } \mathbb{E}[y - w] \geq 0. \end{aligned}$$

Note that the non-negative expected profits restriction must bind in equilibrium: if a principal made positive expected profits in equilibrium, another principal could outbid the "equilibrium" contract by offering  $\delta$  more to the agent in every state of the world, attract the agent, and earn strictly positive expected profits.

Let  $\alpha(s) \in \{0, 1\}$  denote the investment rule implemented following announcement  $s$  from the agent. As we will see below, in a common-priors setting, the information asymmetry does not pose an incentive problem, so that the efficient investment rule  $\alpha^*(s)$  (which is optimal from the principal's perspective) is implemented in equilibrium. Investment is efficient if and only if the project yields positive expected profits conditional on the realization of the signal. That is,  $\alpha^*(s) = 1$  if and only if

$$\mathbb{E}[y | s] \geq 0,$$

which implies

$$s + \int \varepsilon dF(\varepsilon) \geq 0,$$

and thus

$$s + \mu \geq 0.$$

The efficient investment rule can also be expressed as a "hurdle rate"  $s^*(\mu)$ , so that the project is undertaken if and only if  $s \geq s^*(\mu)$ .  $s^*(\mu)$  satisfies

$$s^*(\mu) + \mu = 0.$$

It follows that the equilibrium contract solves

$$\begin{aligned} & \max_{s^*(\mu), w} \mathbb{E}[u(w)] \\ \text{s.t. } & \mathbb{E}[w] = (\mathbb{E}[s \mid s \geq s^*(\mu)] + \mu) \cdot \Pr(s \geq s^*(\mu)). \end{aligned}$$

When principal and agent agree on their beliefs regarding the distribution of  $\varepsilon$ , a fixed-wage contract is the equilibrium contract. The agent receives the full expected productivity of his private information, and, given that he is fully insured, there is never an incentive for him to withhold information (i.e. to “veto” an investment decision). In fact, failing to report  $s$  over a non-measure zero subset of signals when the expected profits from investment conditional on  $s$  are strictly positive would reduce the agent’s payment in equilibrium. In what follows, I assume that an agent who is indifferent between reporting and not reporting will choose to report.

We will subsequently allow for the possibility that principal and agent hold heterogeneous beliefs about the distribution of  $\varepsilon$ . In particular, consider the case in which the agent overestimates his expected productivity. Let a tilde denote the agent’s beliefs: the agent holds the prior belief that  $\varepsilon$  has a cumulative distribution function  $\tilde{F}$  with mean  $\tilde{\mu}$ . We will say that the agent is *overconfident* in this setting if

$$\tilde{\mu} > \mu,$$

which means that an overconfident agent overestimates the expected return of an investment project, conditional on any given signal, relative to the principals’ beliefs.<sup>6</sup>

### 3 Expert Advice

To isolate the consequences of heterogeneous beliefs on the contracted payment schedule from the issue of control (the negotiated investment rule), assume for now that the principal always implements the investment rule which is efficient according to her beliefs. The equilibrium investment

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<sup>6</sup>There are other dimensions over which the beliefs of principal and agent could differ. One could think about an agent who is “overconfident about the precision of  $\varepsilon$ .” The effects of this type of overconfidence will become apparent when we characterize the equilibrium contract allowing for agent and principal to evaluate the likelihood of a given realization of  $\varepsilon$  according to different distribution functions in Section 3. The agent could also be overconfident about the distribution of  $s$ . In this case the agent’s payment would depend on  $s$  in equilibrium. He would be willing to trade a higher payment under those realizations of  $s$  that he believes to be more likely than the principal does, in return for lower payment under those realizations of  $s$  that he believes to be relatively less likely. Absent heterogeneous beliefs about the distribution of  $\varepsilon$ , full insurance with respect to the realization of  $\varepsilon$  will still be optimal. I abstract from the possibility of disagreement about the distribution of  $s$  because it would necessarily distort the principal’s objective function away from efficiency when choosing  $\alpha(s)$ , adding one more source of distortion to the two we study in Section 4.

rule  $\alpha^*(s)$  and respective hurdle rate  $s^*(\mu)$  thus remain unchanged from the common-priors setting discussed in Section 2 above. Note that this investment rule seems too conservative from the agent's point of view: for some realizations of  $s$ , the principal chooses not to invest even though the project is profitable according to the agent's beliefs. If the agent's remuneration depends on the profitability of projects that are undertaken in equilibrium, it is possible that he would be willing to "pay" the principal in exchange for some decision power. We will address this question in Section 4 next.

Recall that the equilibrium contract solves

$$\begin{aligned} & \max \tilde{\mathbb{E}} [u(w)] \\ \text{s.t. } & \mathbb{E}[y - w] \geq 0, \end{aligned}$$

where the tilde over the first expectations operator points to the fact that the agent is interested in maximizing his perceived expected utility, which depends on his beliefs rather than the principal's. As noted before, the non-negative expected profits constraint binds, so we can write

$$\mathbb{E}[w] = (\mathbb{E}[s \mid s \geq s^*(\mu)] + \mu) \cdot \Pr(s \geq s^*(\mu)).$$

Recall that the agent can choose not to report his private information to the principal. In the case of common priors, full insurance for the agent implies that the agent will always reveal his information in equilibrium. If the agent is to fully reveal his information, it must be the case that

$$\tilde{\mathbb{E}}[u(w)] \mid_{\alpha(s)=1} \geq \tilde{\mathbb{E}}[u(w)] \mid_{\alpha(s)=0}$$

whenever the agent observes an  $s$  such that  $\alpha(s) = 1$ . This follows from the assumption in Holmstrom and Ricart i Costa (1986) that no investment is made following no report (i.e. no recommendation to invest) from the agent. As noted by these authors, the principal will in fact decide not to invest following no report from the agent in equilibrium if there are sufficiently "bad" projects. The contract must be such that the agent does not veto a project whenever the equilibrium investment rule dictates investment. This *full-revelation constraint* may or may not bind in equilibrium.

As the following remark notes, in a heterogeneous-beliefs setting the agent will bear some risk following a decision to invest.

**Remark 1** *If  $\tilde{\mu} > \mu$ , then full insurance to the agent is not an equilibrium.*

A formal proof is in the appendix. The intuition is that an overconfident agent is willing to wager against the principal on investment outcome. Consider two potential equilibrium contracts:

one that offers full insurance to the agent, and an alternative contract under which the agent receives higher payment following a high realization of revenue (e.g. profit-sharing, or an option on a share of the profits). Recall that equilibrium requires that the principal receive zero profits in expectation. Therefore, both contracts must yield the same expected profits—and the same expected payment to the agent—*according to the principal's beliefs*. Given that an overconfident agent overestimates his expected productivity, he believes that the risky contract gives him a higher expected payment. This constitutes a first-order gain in terms of his perceived expected utility, while the loss that the agent incurs from bearing risk is only of second order when evaluated at full insurance. It follows that an overconfident agent, even though risk averse, will always be exposed to some risk in equilibrium.

In general, a contract offer will be characterized by an investment rule  $\alpha(s)$ , and payment to the agent  $w(s, \alpha(s), \varepsilon)$ . We can restrict attention, without loss of generality, to a wage function of the form

$$w = \beta + \sigma(\varepsilon),$$

noting that  $\varepsilon$  will be realized only if the project is undertaken (if  $\alpha(s) = 1$ ). Given that  $\varepsilon$  cannot be observed if no investment is made,  $w(s, \alpha(s), \varepsilon) = w(s', \alpha(s'), \varepsilon)$  whenever  $\alpha(s) = \alpha(s') = 0$ , so we can write  $w = \beta$  whenever no investment is made. It is also true that  $w(s, \alpha(s), \varepsilon) = w(s', \alpha(s'), \varepsilon)$  for all  $\varepsilon$  (almost everywhere) whenever  $\alpha(s) = \alpha(s') = 1$ . This follows from the agent's risk aversion, independence between  $s$  and  $\varepsilon$ , and the fact that principal and agent agree about the distribution of  $s$ . Intuitively, it is costless for the principal to insure the agent against variability in  $s$ , and given that principal and agent disagree about the distribution of profits only insofar as they disagree about the distribution of  $\varepsilon$ , it is optimal for the risk-neutral principal to absorb all the risk from variability in  $s$  following investment.<sup>7</sup>

Taking this into account, we can rewrite the principal's zero-expected-profits condition as

$$\begin{aligned} \beta &= \int_{s^*(\mu)}^{\infty} \int (s + \varepsilon - \sigma(\varepsilon)) dF(\varepsilon) dN(s) \\ &= \left( \mathbb{E}[s \mid s \geq s^*(\mu)] + \mu - \int \sigma(\varepsilon) dF(\varepsilon) \right) \cdot \Pr(s \geq s^*(\mu)). \end{aligned}$$

In equilibrium,  $\sigma(\cdot)$  is Pareto optimal from the perspective of securities trading under uncer-

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<sup>7</sup>To see why, consider the following example. For simplicity, imagine that  $s$  and  $\varepsilon$  are discretely distributed. Consider two realizations of the signal,  $s'$  and  $s''$ , such that  $\Pr(s') > 0$ ,  $\Pr(s'') > 0$ . Assume that  $w(s'', 1, \varepsilon') > w(s', 1, \varepsilon')$  for some realization  $\varepsilon'$  of  $\varepsilon$  such that  $\Pr(\varepsilon') > 0$  according to both the principal's and the agent's beliefs. If the principal offers more insurance to the agent in an actuarially fair manner (reducing  $w(s'', 1, \varepsilon')$  and increasing  $w(s', 1, \varepsilon')$  marginally so that expected profits to the principal remain constant), the risk-averse agent's perceived expected utility will increase, independent of what principal and agent believe to be the actual likelihood of  $\varepsilon'$ .

tainty given heterogeneous beliefs:

$$u'(\beta + \sigma(\varepsilon)) \frac{\tilde{f}(\varepsilon)}{f(\varepsilon)} = \lambda \quad \forall \varepsilon,$$

where  $\lambda$  is some constant. The agent receives a higher payment following realizations of  $\varepsilon$  that he believes are more likely than the principal does. In the extreme case of a risk-neutral agent (and principal), the agent would hold an infinitely “long” position in those states he believes are strictly more likely than the principal does, and an infinitely “short” position in those states he believes are strictly less likely than the principal does. The agent’s risk aversion limits the extent to which he is willing to wager against the principal, however. It is interesting to note that if the agent believes that high realizations of  $\varepsilon$  are more likely than the principal does, then the equilibrium contract will resemble an incentive contract (in the sense that the agent is paid more for higher realizations of profit) even though there is no incentive problem intrinsic to the agency relationship.

If the full-revelation constraint is slack in equilibrium, then we can further state that

$$u'(\beta + \sigma(\varepsilon)) \frac{\tilde{f}(\varepsilon)}{f(\varepsilon)} = u'(\beta).$$

Rearranging this expression and integrating with respect to  $\varepsilon$ , we find that

$$\int u'(\beta + \sigma(\varepsilon)) \tilde{f}(\varepsilon) d\varepsilon = u'(\beta),$$

or

$$\tilde{\mathbb{E}}[u'(\beta + \sigma(\varepsilon))] |_{\alpha(s)=1} = u'(\beta).$$

The intuition behind this result is that, because agent and principal do not disagree about the distribution of the signal  $s$ , the agent is offered as much insurance as possible (in terms of his marginal utility of consumption) across the “invest” and “do not invest” events, which depend solely on the realization of  $s$  when the full-revelation constraint does not bind.

If, on the other hand, the full-revelation constraint binds in equilibrium, then it must be the case that

$$\int u(\beta + \sigma(\varepsilon)) \tilde{f}(\varepsilon) d\varepsilon = \tilde{\mathbb{E}}[u(\beta + \sigma(\varepsilon))] |_{\alpha(s)=1} = u(\beta)$$

and

$$u'(\beta + \sigma(\varepsilon)) \frac{\tilde{f}(\varepsilon)}{f(\varepsilon)} = \kappa \quad \forall \varepsilon$$

for some constant  $\kappa$  such that  $\kappa \leq u'(\beta)$ . A risk-averse agent who holds  $\varepsilon$ -risk in equilibrium might need a “bonus for investment,” given that he can guarantee himself  $u(\beta)$  by never reporting. Under heterogeneous beliefs, the size of the bonus depends on the realization of  $\varepsilon$ , so as to provide as much insurance as possible to the agent (again, in terms of the agent’s marginal utility of consumption)

after accounting for the wagering between principal and agent because of their disagreement about the distribution of  $\varepsilon$ . The full-revelation constraint might bind in equilibrium because a risk-averse agent who overestimates his expected payment conditional on investment being made would like to receive a high payment conditional on investment not being made (the insurance motive across the “invest” and “do not invest” events) when agent and principal agree about the distribution of the signal  $s$ .<sup>8</sup>

These results are summarized in the following proposition.

**Proposition 1** *Under heterogeneous beliefs, and assuming that  $s^*(\mu)$  is the hurdle rate guiding investment decisions, in equilibrium the agent receives  $\varepsilon$ -contingent payment  $w = \beta + \sigma(\varepsilon)$ , where*

1.  $\beta = \int_{s^*(\mu)}^{\infty} \int (s + \varepsilon - \sigma(\varepsilon)) dF(\varepsilon) dN(s)$ , and
2.  $u'(\beta + \sigma(\varepsilon)) \frac{\tilde{f}(\varepsilon)}{f(\varepsilon)} = \lambda \forall \varepsilon$ , where  $\lambda$  is some constant.
  - $\lambda = u'(\beta)$ , so that  $\tilde{\mathbb{E}}[u'(\beta + \sigma(\varepsilon))] |_{\alpha(s)=1} = u'(\beta)$ , if the full-revelation constraint is slack.
  - $\lambda \leq u'(\beta)$  and  $\tilde{\mathbb{E}}[u(\beta + \sigma(\varepsilon))] |_{\alpha(s)=1} = u(\beta)$  if the full-revelation constraint binds.

The proof follows immediately from constrained maximization of the agent’s perceived expected utility, subject to zero expected profits for the principal and the full-revelation constraint.

## 4 Control

Throughout the previous section, we assumed that the implemented investment rule was the efficient rule according to the principal’s beliefs,  $\alpha^*(s)$ . From the agent’s point of view, this investment rule is suboptimal. Given that the agent’s payment depends only on the realization of  $\varepsilon$ , the realization of  $s$  does not directly affect his utility. The investment rule that is implemented, however, affects the principal’s evaluation of expected revenue and thus expected payment to the agent. The investment rule therefore affects the agent’s perceived expected utility after accepting a given contract, so  $\alpha^*(s)$  might not be implemented in equilibrium.

If principal and agent disagree about expected revenue following investment conditional on a given signal  $s$ , they will disagree about which is the efficient investment rule. As discussed before, after observing signal  $s$  it is efficient to invest in the project whenever expected revenue

<sup>8</sup>If that was not the case, they would wager on the realization of  $s$  just like they wager on the realization of  $\varepsilon$ , so that the full-revelation constraint is more likely to be slack if the agent overestimates the likelihood of receiving “good” signals that lead to investment.

from investing, conditional on the available information, is non-negative. This condition is met whenever

$$s + \mu \geq 0$$

according to the principal's beliefs, and whenever

$$s + \tilde{\mu} \geq 0$$

according to the agent's beliefs. In the case of agent overconfidence, so that  $\tilde{\mu} > \mu$ , there will be a range of signals over which principal and agent disagree about whether or not it is efficient to invest. Principal and agent agree that it is efficient to invest whenever  $s \geq -\mu$ , and that it is efficient not to invest whenever  $s < -\tilde{\mu}$ . For those realizations of  $s$  such that  $-\tilde{\mu} \leq s < \mu$ , however, the principal believes that it is best not to invest, while the agent believes that it is efficient to invest. Under heterogeneous beliefs, payment to the agent depends on the realization of  $\varepsilon$  following investment. It follows that the implemented investment rule may affect the agent's evaluation of his expected utility. We characterize the equilibrium contract when the investment rule is contractible in the following proposition.

**Proposition 2** *The equilibrium investment rule can be characterized by a "hurdle rate"  $\hat{s}(\mu)$ , so that  $\alpha(s) = 1$  if and only if  $s \geq \hat{s}(\mu)$  and  $\hat{s}(\mu)$  is implicitly defined by*

$$\hat{s}(\mu) + \mu - \int \sigma(\varepsilon) dF(\varepsilon) + \frac{1}{u'(\beta)} \left\{ \int u(\beta + \sigma(\varepsilon)) d\tilde{F}(\varepsilon) - u(\beta) \right\} = 0,$$

where  $\beta$  and  $\sigma(\varepsilon)$  are defined as in Proposition 1 substituting  $\hat{s}(\mu)$  for  $s^*(\mu)$ .

The proof follows immediately from including the hurdle rate  $\hat{s}(\mu)$  as a control variable in the problem of maximizing the agent's perceived expected utility, subject to zero expected profits for the principal and the full-revelation constraint.

First, note that the principal finds it in her best interest to implement what she believes to be the efficient rule only if she retains full stake (on average) in the project. In the case of common priors, given that the agent is offered full insurance in equilibrium, the principal receives the full marginal benefit from investing. Because of this, and given that the principal is risk neutral, her objective function is aligned with efficiency. The investment rule implemented in equilibrium is thus the efficient investment rule. If the principal relinquishes some stake in the project to the agent, the optimal investment rule from the principal's point of view will no longer be the one that she considers socially efficient.

Consider the investment rule that the principal would implement if she had full control over the investment decision, and she could not commit *ex-ante* to implementing any given investment rule.

This would imply that the principal could not negotiate with the agent about the investment rule; absent commitment, the principal will implement the investment rule that maximizes her expected profits conditional on  $s$ . Because in equilibrium expected profits are zero for the principal, this investment rule maximizes the fixed payment  $\beta$  to the agent, thus maximizing the agent's perceived expected utility. Recall that zero expected profits for the principal implies

$$\beta = \int_{\hat{s}(\mu)}^{\infty} \int (s + \varepsilon - \sigma(\varepsilon)) dF(\varepsilon) dN(s)$$

given hurdle rate  $\hat{s}(\mu)$ . Let  $s^{*P}(\mu)$  denote the optimal hurdle rate from the principal's point of view—which allows her to maximize the fixed payment  $\beta$  to the agent—implicitly defined by

$$\frac{d\beta}{d\hat{s}(\mu)} = - \int (s^{*P}(\mu) + \varepsilon - \sigma(\varepsilon)) dF(\varepsilon) = 0.$$

In the case of common priors, the agent is fully insured so that  $\sigma(\varepsilon) = 0$  for all  $\varepsilon$ , and the above expression simply yields  $s^{*P}(\mu) = s^*(\mu) = -\mu$ . If the principal gives some positive- or negative-expected value stake in the project to the agent, so that  $\int \sigma(\varepsilon) dF(\varepsilon) \neq 0$ , it follows that  $s^{*P}(\mu) \neq s^*(\mu)$ . Intuitively, when the principal relinquishes some of her claim over the project's outcome, the optimal investment rule according to her beliefs skews away from efficiency. Imagine, for instance, the extreme case in which the principal gives all the marginal benefits from the project to the agent and simply insures him against variation in  $s$ , so that  $\sigma(\varepsilon) = \varepsilon$ . Ex-post, the principal would like to invest only whenever her share in the project yields positive profits, so she would set  $s^{*P}(\mu) = 0$ . This effect is of particular interest in settings where the agent's payment is bounded below (e.g. limited liability, or if the agent's remuneration includes an option on firm value).

There is a second source of distortion in the equilibrium investment rule, which arises only when the full-revelation constraint is slack. In that case,  $\int u(\beta + \sigma(\varepsilon)) d\tilde{F}(\varepsilon) > u(\beta)$  in equilibrium, so that the agent believes he is strictly better off following a decision to invest than when the contract calls for no investment. As a consequence, the agent *de facto* gives up some of his fixed payment  $\beta$  in return for a more liberal investment decision rule than  $s^{*P}(\mu)$ . Consider the particular case in which the principal retains full stake in the project, so that  $\int \sigma(\varepsilon) dF(\varepsilon) = 0$  in equilibrium and  $s^{*P}(\mu) = s^*(\mu)$ ; there is no distortion of the principal's objective away from efficiency. If the principal can commit to implementing a given investment rule, and  $\int u(\beta + \sigma(\varepsilon)) d\tilde{F}(\varepsilon) > u(\beta)$ , the equilibrium investment rule will be  $\hat{s}(\mu) < s^*(\mu)$ .

A risk-averse overconfident agent who is not fully insured believes that he *does* hold positive stake in the project. Equilibrium requires this to be the case: given that the agent is not fully insured, risk aversion and the full-revelation constraint require that  $\int \sigma(\varepsilon) d\tilde{F}(\varepsilon) > 0$ . We cannot, however, say anything about the sign of  $\int \sigma(\varepsilon) dF(\varepsilon)$ : given that the agent is paid more (a higher

$\sigma(\varepsilon)$ ) under those realizations that he believes more likely relative to the principal's beliefs, it follows that  $\int \sigma(\varepsilon) d\tilde{F}(\varepsilon) > \int \sigma(\varepsilon) dF(\varepsilon)$ .

In the case of common priors, risk aversion implies that  $\int u(\beta + \sigma(\varepsilon)) dF(\varepsilon) < u(\beta)$  whenever  $\int \sigma(\varepsilon) dF(\varepsilon) = 0$  and  $\sigma(\varepsilon) \neq 0$  for some non-measure-zero subset of possible realizations of  $\varepsilon$ . This is why full insurance is optimal in that case, and the efficient investment rule—which is optimal from the principal's point of view—is implemented in equilibrium. Full revelation follows trivially from full insurance to the agent. In the case of agent overconfidence, given that the risk-averse agent is not fully insured, the full-revelation constraint and risk aversion require that  $\int \sigma(\varepsilon) d\tilde{F}(\varepsilon) > 0$ . If the agent is sufficiently overconfident (or has enough risk tolerance), this will imply that  $\int u(\beta + \sigma(\varepsilon)) d\tilde{F}(\varepsilon) > u(\beta)$  in equilibrium. The principal will then give some degree of control to the agent, in the sense that the investment rule implemented in equilibrium will be skewed away from what the principal would unilaterally choose towards a more liberal investment rule.

## 5 Example

Consider the particular case of an exponential utility function, so that the agent's utility can be expressed as

$$u(w) = -\exp^{-rw},$$

and normally-distributed productivity shocks  $\varepsilon$ . According to the principal's beliefs,  $\varepsilon$  has mean  $\mu_\varepsilon$ , and mean  $\tilde{\mu}_\varepsilon > \mu_\varepsilon$  according to the agent's beliefs; assume that both participants believe that  $\varepsilon$  is distributed with precision  $h_\varepsilon$ .

It is useful to note that the full-revelation constraint will not be binding in equilibrium (although it will be satisfied with equality) when the agent has an exponential utility function. Recall that if the full-revelation constraint does not bind, then because of the insurance motive

$$\int u'(\beta + \sigma(\varepsilon)) d\tilde{F}(\varepsilon) = u'(\beta).$$

The agent wishes to smooth his marginal utility of consumption across “invest” and “do not invest” events.<sup>9</sup> If the agent's utility function is exponential, we have that

$$u'(w) = r \exp^{-rw},$$

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<sup>9</sup>As noted before, this result follows from agent and principal holding identical beliefs regarding the distribution of the signal  $s$ .

which is simply a linear transformation of  $u(w)$ . Smoothing marginal utility thus also implies smoothing total utility for the agent:

$$\int u(\beta + \sigma(\varepsilon)) d\tilde{F}(\varepsilon) = u(\beta)$$

where  $\beta$  and  $\sigma(\varepsilon)$  maximize the agent's perceived expected utility subject only to non-negative expected profits for the principal. Therefore, the full-revelation constraint does not bind in equilibrium, and it is satisfied with equality.

The equilibrium contract is thus characterized by

$$u'(\beta + \sigma(\varepsilon)) \frac{\tilde{f}(\varepsilon)}{f(\varepsilon)} = u'(\beta).$$

Solving for  $\sigma(\varepsilon)$  we find

$$\sigma(\varepsilon) = \frac{1}{r} (\tilde{\mu}_\varepsilon - \mu_\varepsilon) h_\varepsilon \left[ \varepsilon - \frac{\tilde{\mu}_\varepsilon + \mu_\varepsilon}{2} \right],$$

a linear, increasing function of  $\varepsilon$ . As should be expected, the degree of "wagering" on the realization of  $\varepsilon$  increases with the degree of disagreement between principal and agent regarding the mean of  $\varepsilon$  ( $\tilde{\mu}_\varepsilon - \mu_\varepsilon$ ), with the precision of  $\varepsilon$  ( $h_\varepsilon$ ), and also with the agent's tolerance for risk (measured in this case by  $\frac{1}{r}$ ). The average of the two means,  $\frac{\tilde{\mu}_\varepsilon + \mu_\varepsilon}{2}$ , is the one realization of  $\varepsilon$  for which  $\tilde{f}(\varepsilon) = f(\varepsilon)$ ;  $\tilde{f}(\varepsilon) > f(\varepsilon)$  for any realization of  $\varepsilon$  above that, and  $\tilde{f}(\varepsilon) < f(\varepsilon)$  for any realization below that.

In this particular case, we also find that

$$\int \sigma(\varepsilon) dF(\varepsilon) = - \int \sigma(\varepsilon) d\tilde{F}(\varepsilon) = -\frac{1}{r} h_\varepsilon \frac{(\tilde{\mu}_\varepsilon - \mu_\varepsilon)^2}{2} < 0.$$

Given that  $\int \sigma(\varepsilon) dF(\varepsilon) < 0$ , we know that the equilibrium investment rule is more liberal than what the socially efficient rule would be according to the principal's beliefs. Even though the agent is indifferent in equilibrium between "invest" and "do not invest" events, the contract is such that the principal expects to pay less to the agent on average in the event that an investment is made than if no investment is made. She will therefore choose to invest in some instances when she expects the financial returns to investment to be negative, because they are overturned by savings in terms of lower expected payment to the agent.

Consider now the case of agent overconfidence regarding the precision of  $\varepsilon$  rather than its mean. Assume that the principal believes  $\varepsilon$  to be normally distributed with mean  $\mu_\varepsilon$  and precision  $h_\varepsilon$ , while the agent believes it to have mean  $\mu_\varepsilon$  and precision  $\tilde{h}_\varepsilon > h_\varepsilon$ . In this case, in equilibrium

$$\sigma(\varepsilon) = \frac{1}{r} \left( \ln \tilde{h}_\varepsilon - \ln h_\varepsilon \right) - \frac{1}{2r} \left( \tilde{h}_\varepsilon - h_\varepsilon \right) (\varepsilon - \mu_\varepsilon)^2.$$

Intuitively, an agent who overestimates the precision of  $\varepsilon$  relative to the principal will be willing to accept a lower payment following extreme realizations of  $\varepsilon$  in exchange for higher payment following realizations of  $\varepsilon$  close to  $\mu_\varepsilon$ .

## 6 Concluding Remarks

When principal and agent disagree about the distribution of profits following investment, the risk-averse agent bears project risk in equilibrium. Because the agent is not fully insured in equilibrium, there are two sources of distortion of the implemented investment rule away from what the principal considers the efficient investment rule. First, if payments to the agent are higher (or lower) in expectation following a decision to invest than when no investment is made according to the principal's beliefs, the principal's objective function is skewed away from efficiency. In the example in Section 5, the principal expected to pay less to the agent under those events in which investment was undertaken. As a consequence, the optimal investment rule from the principal's point of view dictated for investment in some situations in which she believed investment to be inefficient. The second source of distortion comes from the agent's evaluation of his payments. Because of the full-revelation constraint, it must be the case that the agent's perceived expected utility following investment is at least as high as his expected utility when no investment is made. If this constraint is slack in equilibrium, so that the agent believes his expected utility following investment is strictly higher than when no investment is made, then he will be willing to "pay" the principal to implement investment more often. This could be interpreted as the principal giving the agent some degree of control over the investment decision, even though they both disagree about the best course of action.

This result is of some interest in the context of previous literature. It has similar implications to those pointed out by Gervais, Heaton and Odean (2003) and Van den Steen (2005): in the presence of agent overconfidence, pay-for-performance incentive contracts create distortions in the allocation rules implemented in an agency relationship. One important point of this paper, however, is that pay-for-performance is a direct consequence of the disagreement between principal and agent, as shown in Section 3. One way to approach the intuition is that the principal, when designing the contract, is faced with a dilemma: higher-powered incentives introduce a costly distortion in terms of efficiency, but they result in savings in terms of expected payment to the agent. The conclusion that decision rules are distorted away from efficiency (according to the principal's beliefs) need not be troubling. If the agents tend to "get it right" more often than the principals, there are actual efficiency gains as the principal relinquishes some control over to the agent. In light of the literature on overconfidence, however, moderate optimistic bias from the agent seems more likely than bias from the principal.

Investment advice is obtained by firms both from outsiders (e.g. a consultant), and insiders (e.g. an in-house expert). Because of the type of contracts that tend to characterize consulting relationships, there are more constraints on the pay to outsiders as to the extent that pay-for-

performance can depend on outcomes (both in terms of size and time period). The results outlined in this paper imply that the expected profit (before subtracting payment to the agent) of projects undertaken following outsiders' expert advice should be higher than the expected profit following insiders' expert advice. In first light, one could (erroneously) infer that outsiders tend to be more knowledgeable than insiders in terms of investment decisions from such asymmetry. We would then observe that outsiders would be hired even in some situations in which an insider is better informed to provide advice. Further research in this area could yield insight into corporate decision-making structures.

A natural extension to the model is to consider the effects of heterogeneous beliefs in the presence of career concerns. In a multi-period setting, and when the agent cannot bind himself to the agency relationship (i.e. when the agent cannot commit not to quit and be employed by another principal in subsequent periods), he will necessarily bear some reputational risk: investment outcome is informative about the agent's ability, and this information will be incorporated into competing principal's contract offers after the first period. A risk-averse agent might therefore be reluctant to share his private information in early periods. An agent who is overconfident about his ability relative to the principals' beliefs, on the other hand, believes that they are likely to update their beliefs upwards following investment. An overconfident agent therefore overestimates the benefits of first-period investment in terms of his second-period remuneration. Because of this, overconfidence is likely to mitigate the agency problems related to agent risk aversion in a career-concerns setting, even though an overconfident agent bears more risk in equilibrium. This issue is studied in de la Rosa (2006b), available upon request from the author.

## A Appendix

**Remark 1** *If  $\tilde{\mu} > \mu$ , then full insurance to the agent is not an equilibrium.*

**Proof.** As a simplification, restrict attention to a linear profit-sharing contract of the form

$$w = \beta + \sigma y.$$

If full insurance is the equilibrium contract, then  $\sigma = 0$  at the optimum.<sup>10</sup>

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<sup>10</sup>Profit sharing might not be the optimal contract. For instance, an option on the value of the firm could provide some insurance to the agent, and reduce the cost to the principal of hiring the agent. It is sufficient to show, however, that a profit-sharing contract dominates a full-insurance contract.

Expected payment to the agent is thus

$$\begin{aligned}\mathbb{E}[w] &= \beta + \sigma \mathbb{E}[y \mid \mu, \alpha^*(s)] \\ \mathbb{E}[y \mid \mu, \alpha^*(s)] &= \beta + \sigma \mathbb{E}[y \mid \mu, \alpha^*(s)].\end{aligned}$$

Solving for  $\beta$  yields

$$\beta = (1 - \sigma) \mathbb{E}[y \mid \mu, \alpha^*(s)],$$

where  $\mathbb{E}[y \mid \mu, \alpha^*(s)] = (\mathbb{E}[s \mid s \geq s^*(\mu)] + \mu) \cdot \Pr(s \geq s^*(\mu))$ .

The equilibrium contract therefore solves

$$\begin{aligned}\max_{\sigma} \tilde{\mathbb{E}}[u(\beta + \sigma y)] \\ \text{s.t. } \beta &= (1 - \sigma) \mathbb{E}[y \mid \mu, \alpha^*(s)].\end{aligned}$$

Note

$$\begin{aligned}\tilde{\mathbb{E}}[u(\beta + \sigma y)] &= \int_0^{s^*(\mu)} u((1 - \sigma) \mathbb{E}[y \mid \mu, \alpha^*(s)]) dN(s) \\ &\quad + \int_{s^*(\mu)}^{\infty} \int u((1 - \sigma) \mathbb{E}[y \mid \mu, \alpha^*(s)] + \sigma(s + \varepsilon)) d\tilde{F}(\varepsilon) dN(s).\end{aligned}$$

Evaluating the derivative of the objective function with respect to the agent's share of revenue  $\sigma$  at  $\sigma = 0$  yields

$$\begin{aligned}\left. \frac{d}{d\sigma} \tilde{\mathbb{E}}[u(\beta + \sigma y)] \right|_{\sigma=0} &= u'(\beta) \int_0^{s^*(\mu)} -\mathbb{E}[y \mid \mu, \alpha^*(s)] dN(s) \\ &\quad + u'(\beta) \int_{s^*(\mu)}^{\infty} \int (s + \varepsilon - \mathbb{E}[y \mid \mu, \alpha^*(s)]) d\tilde{F}(\varepsilon) dN(s) \\ &= u'(\beta) \left( \tilde{\mathbb{E}}[y \mid \tilde{\mu}, \alpha^*(s)] - \mathbb{E}[y \mid \mu, \alpha^*(s)] \right) \\ &= u'(\beta) [(\tilde{\mu} - \mu) \cdot \Pr(s \geq s^*(\mu))] > 0.\end{aligned}$$

It follows that, if restricted to linear profit sharing, the agent will hold some stake in the project in equilibrium. Full insurance is, therefore, not an equilibrium. ■

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