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Spectral Analysis of GDP shocks in US and BRIC counties

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Abstract : The aim of the article is to show that Spectral Analysis can be applied to the real business cycle analysis in US and BRIC counties. The paper will use long time series of data for US and BRIC countries to investigate the evolution of the GDP shocks and if the fractal wave model can explain better the volatility of the shocks than the traditional regression models.
The empirical analysis consists of four steps: In the first step aggregate fluctuations are obtained by using Hodrick-Prescott Filter. In the next three steps fluctuations are described by using ARMA (2, 2) model. Estimated ARMA parameters are than used to perform spectral analysis and to calculate the parameters of the business cycle which appears to be a damped sine wave. Amplitude , frequency and phase of the sine waves are than determined using Spectral harmonic analysis based on autoregressive model .The results show evidence that GDP shocks can be modeled and forecasted using multi step harmonic analysis .All empirical results refer to the data taken from ECOWIN for period :1980-2009 . They are obtained using E-Views and MATLAB.

Introduction

Having a better model for GDP shocks should lead to better policymaking, which should, in turn, mitigate the positive and negative deviations from the trend, and maintain the proper direction and magnitude of growth in the GDP. Economic theory has moved towards the study of economic fluctuation rather than a 'business cycle' - though they use the phrase 'business cycle' as convenient shorthand. Thus, it is well accepted in the literature that the cycle involves shifts over time between periods of relatively rapid growth of output (recovery and prosperity), alternating with periods of relative stagnation or decline (contraction or recession). Agregate fluctuations are often measured using the real gross domestic product or gross national product . According to some acconomist ,to call those alternances "cycles" is rather misleading as they don't tend to repeat at fairly regular time intervals. Most observers find that their lengths (from peak to peak, or from trough to trough) vary, so that cycles are not mechanical in their regularity. Since no two cycles are alike in their details, some economists dispute the existence of cycles and use the word "fluctuations" , as Prescott did.
Nonetheless there area two schools of thought about periodical behavior of the real economic output. The initial thoughts are related to the wave theory.

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The main types of business cycles are enumerated by Joseph Schumpeter and others in this field, have been named after their discoverers: the Kitchin inventory cycle (3-5 years) - after Joseph Kitchen; the Juglar fixed investment cycle (7-11 years) -- after Clement Juglar; the Kuznets infrastructural investment cycle (15-25 years) -- after Simon Kuznets, Bronson Asset Allocation ~30 ;Nobel Laureate; the Kondratieff wave or cycle (45-60 years) -- after Nikolai Kondratieff;.
In the Juglar cycle,recovery and prosperity are associated with increases in productivity, consumer confidence, aggregate demand,and prices.
According to the Kondratiev's innovation theory, these waves arise from the bunching of basic innovations that launch technological revolutions that in turn create leading industrial or commercial sectors.
However, the Austrian School, of economics, following Ludwig von Mises, rejects the suggestion that the business cycle is an inherent feature of an unregulated economy and argues that it is caused by intervention in the money supply.

Elliott established his wave principle as "the secret of the universe," and said that "because man is subject to rhythmical procedure, calculations having to do with his activities can be projected far into the future with a justification and certainty heretofore unattainable". An important feature of Elliott Wave is that they are fractal in nature. 'Fractal' means market structure is built from similar patterns on a larger or smaller scales. Therefore, we can count the wave on a long-term yearly market chart as well as short-term hourly market chart. Elliot's approach is ad hoc or intuitive in nature.
So far there was no scientific analysis of his theory. In the area of business and economics, a key question is whether or not there are similar *mechanisms* that generate recessions and/or booms that exist in economies so that the dynamics that appear as a cycle will be seen again and again.

The objective of this paper is to show that GDP cycles exist; that cycles are fractal in Elliot's sense and that they can be modeled by a sequence of filters which extract signals with progressive frequencies. The empirical analysis consists of four steps: In the first step aggregate fluctuations are obtained by using Hodrick-Prescott Filter. In the next steps fluctuations are models by ARMA (2,2) model . Estimated ARMA parameters are used to calculate the parameters of the internal cycle which appears to be a damped sine wave. Amplitude , frequency and phase of the sine waves are than determined using harmonic analysis based on ARMA models .The results show evidence that GDP shocks can be modeled and forecasted using two step harmonic analysis . In this context H-P filter is a special case of the wave extraction filter based on ARMA filter.

The standard Real Business Cycle model (RBC), as introduced by Kydland and Prescott (1982), challenged the dominant view that business cycles, or more precisely aggregate fluctuations, are caused by monetary and financial shocks. According to that view (Hawtrey's Pure Money Cycles), upswings in economic activity result from unexpectedly rapid increases in the supply of money, while downswings result from slow growth or a fall in the money supply.
In contrast, Prescott and his collaborators presented evidence that business cycles of the sort seen during the postwar era would occur even if there were no monetary or financial

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disturbances. Prescott argued that aggregate fluctuations of output, consumption, investment and hours worked are driven by technology shocks at the beginning his model was remarkably successful in mimicking cyclical pattern of those variables.
Empirical test of the standard model (Mc Grattan 1994) showed that the standard model can account for the observed variability in output, investment, and capital stock. Despite the successes achieved in accounting for the aggregate quantities, business cycle models have been unable to replicate features of relative prices The main failures of the model were its inability to generate the observed variability in consumption, hours worked, and productivity and its inability to generate a near-zero correlation between hours worked and productivity.

Early attempt to extend that model was made by Hansen (1985) who assumed that households can work a fixed number of hours, N, or none at all .This concept is known as a concept of indivisible labor . Kydland and Prescott (1989) gave a theoretical foundation for the ad hoc concept of indivisible labor introduced by Hansen. They improved slightly a standard RBC model by assuming that both capital utilization and hours per worker vary. They demonstrated that Solow technological shocks explain up to 70% of aggregate output fluctuations.
However, total output could also change if the effectiveness of the workers and equipment used in production changes. . Economists refer to this change in the effectiveness with which workers and machinery generate value-added as a change in total factor productivity (TFP).
Chang (1992) and Braun (1994) have noted that most of the failures of the standard model can be reconciled once fiscal shocks are included in the model. They showed that fiscal shocks can better mimic the observed patterns of aggregate fluctuations such as the variability in consumption, hours worked, and productivity and the near-zero correlation between hours worked and productivity. McGrattan and Prescott (2003) computed annual after-tax returns for the non-corporate sector. Adding fiscal shocks to the standard Kydland-Prescott model significantly improved its ability to mimic the fluctuations of U.S. aggregate data.
Rouwenhorst (1995) has shown that the inter- temporal marginal rate of substitution (IMRS) or the stochastic discount factor is not volatile enough to account for the time series properties of S&P 500 returns. He demonstrated that the basic RBC model captures about 4% of the volatility in S&P 500 returns. As noted by Mulligan (2002) in the context of inter-temporal substitution in consumption, in aggregate models this relative price is not the rate of return on S&P 500 assets. Since tangible assets of S&P 500 firms are a fraction of the aggregate capital (40%), the link between theory and measurement is, at best, weak. Gomme and Rupert (2005) found that this weak link between theory and measurement would not be problematic for quantitative exercises if the empirical properties of the return to capital were the same as those of the S&P 500 returns. They constructed a time series for the return to capital and show that its time series properties differ significantly from those of the S&P 500 returns. Further they showed that the basic RBC model with logarithmic preferences accounts for nearly 40% of the volatility in the return to capital. A model with indivisible labor generates roughly the same relative volatility, whereas a model with home production generates 25% of the relative volatility.

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Empirical analysis:

Step 1: Hodrick-Prescott Filter

Technically, the Hodrick-Prescott (HP) filter is a two-sided linear filter that computes the smoothed series { s_t } of { y_t } by minimizing the variance of { y_t } around { s_t } , subject to a penalty that constrains the second difference of { s_t }. That is, the HP filter chooses to minimize:

$\Sigma (y_t - s_t)2 + \lambda [(s_t - s_{t-1}) - (s_{t-1} - s_{t-2})]^2$ (1)

$\Sigma (y_t - s_t)2 + \lambda [D(s_t)^2]$ (2)

The penalty parameter λ con rols the smoothness of the series . The larger the λ , the smoother the {s} . For the quarterly data Prescott suggested λ=1600

Step 2: Modeling sine wave using ARMA model

It was proved by Yule (1927) that a single period sine wave can be treated as a solution of the second order difference equation:

$\alpha_0 * Y_t - \alpha_1 * Y_{t-1} + \alpha_2 * y_{t-2} = 0$ (3)

The general solution of the equation is

$y = e^{pt}(c1*\cos(qt) + c2*\sin(qt))$ (4)

where :

$p = \alpha 1/2\alpha_0$ and where q is :

$$q1,2 = \frac{\sqrt{(\alpha_1^2 - 4\alpha_0\alpha_2)}}{2\alpha_0}$$ and (5)

and where c1 and c2 are l constants which are to be determined using initial y values.

Difference equation might have real roots : $\alpha_1^2 - 4\alpha_0 * \alpha_2 > 0$ when over dumped periodicals are generated ,complex roots $\alpha_1^2 - 4\alpha_0 * \alpha_2 < 0$ (when under-dumped periodicals are generated) or critical dumping roots when $\alpha_1^2 - 4\alpha_0 * \alpha_2 = 0$

Finally one can easily derive that when the solution has a complex roots (under-dumped periodical motion), Y_t can be represented as a sine wave:

$Y_t = A\sin(\phi)\cos(\omega * t) + \cos(\phi)\sin(\omega * t)$ (5a)

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After some simple trigonometric transformation Yt can be written as :
$Y_t = A * \sin(\omega * t + \phi)$, (6)
where A is the amplitude , $\omega = 2 * \pi * f_m$ is angular frequency in radians, f_m is a carrier frequency and ϕ is the phase shift
Since the second order difference equation corresponds to the autoregressive model AR(2) , parameters of the sine wave can be calculated from the roots of its characteristic equation (Box-Jenkins 1976, pg 545) .This was used for the narrow Band Spread Spectrum Interference Reduction (Dudukovic 1989)
If the optimal model for Yt is AR(2) with imaginary roots of the characteristic equation than frequency, phase and amplitude can be obtained using estimated AR parameters, as following:
$\mathbf{Y}_t = \boldsymbol{\alpha}_1 * \mathbf{Y}_{t-1} + \boldsymbol{\alpha}_2 * \mathbf{Y}_{t-1} + a_t$, (7)
where a_t is a white noise with the variance σ_a^2.
$\boldsymbol{\alpha}_1 = 2\exp(-\delta_s) * \cos(2\pi f_m)$ (8)
$\boldsymbol{\alpha}_2 = -\exp(-2\delta_s)$, (9)
where δ_s is a damping factor .If δ_s is 0 ,equations (5a) and (6) become equal.
If autoregressive signal AR(p) is embedded in white noise, the resulting is ARMA(p,p) signal , as it was proven by Pagano (1974).
Harmonic characteristics of the AR(p) signal can be easily calculated from the estimates of AR parameters of the Mixed ARMA model as was proved by Gucnait J (1986).

Data Description

All empirical results refer to the data taken from ECOWIN for period :1980-2009.Descriptiv statistics for all four countries : Brazil, China, Us, Russia and India is presented in the table 1.

Table 1

	BRGDP	CHGDP	USGDP	RUGDP	INGDP
Mean	3.36E+11	7.11E+12	7.73E+12	8.52E+12	5.12E+11
Median	3.01E+11	5.33E+12	7.77E+12	9.31E+12	4.89E+11
Maximum	7.47E+11	3.01E+13	1.10E+13	1.17E+13	1.31E+12
Minimum	49069000	4.97E+11	4.71E+12	4.87E+12	0
Std. Dev.	2.14E+11	6.11E+12	1.97E+12	2.37E+12	3.83E+11
Skewness	0.190497	1.628312	0.193084	-0.18698	0.280946
Kurtosis	2.151045	5.705134	1.829463	1.639336	2.256856
Jarque-Bera	2.489408	51.52964	4.36794	5.724853	2.495455
Probability	0.288026	0	0.112594	0.05713	0.287157
Sum	2.32E+13	4.90E+14	5.33E+14	5.88E+14	3.53E+13
Sum Sq. Dev.	3.11E+24	2.54E+27	2.63E+26	3.83E+26	9.96E+24
Observations	69	69	69	69	69

Table 1 shows some discrepancy from the skewness and kurtosis parameters of the Gaussian distribution .Hodrick-Prescott filter was the first one to be applied to all the DGP variables. HP trend is than modeled using ARMA (2,2) model. It was proved that HP filter is nothing else than a smooth ARMA(2,2) model or the first fractal wave .

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The table 2 shows that coefficient of determination for HP trend for all the models was almost 100%.

Table 2:

HP Trend models

		AR(1)	AR(2)	MA(1)	MA(2)		R2
INDIA		0.566518	0.481914	1.188523	0.994999		0.999881
RUSSIA		2.019958	-1.020251	1.272495	0.966226		1.00
CHINA		2.01045	-1.010476	1.380564	0.994895		1.00
BRAZIL		1.986934	-0.986452	1.285042	0.994984		1.00
US		2.048768	-1.049358	0.813933	0.961593		1.00

This confirms that Hodrick and Prescott were right when assuming the presence of the smooth trend independent of the remaining cyclical component. However ,they were not aware that that the trend was also cyclical , but almost deterministic.

ARMA coefficients are obtained for the next three fractal waves, using HP residuals , known in the literature as GDP fluctuations, and applying ARMA parameter estimation subsequently thee times . The results are presented in the table three.

Table 3: Fractal Waves-Models

Sine wave models

			AR(1)	AR(2)	MA(1)	MA(2)	R2	Frequency
INDIA	Fractal wave 1	HP	0.5665	0.4819	1.1885	0.9950	99.99%	0.6328
	Fractal wave 2		0.4201	0.5277			89.98%	0.5859
	Fractal wave 3		-0.3931	-0.0694	-0.3077	0.9405	54.41%	0.0021
	Fractal wave 4		0.3237	0.2684	-0.9209	0.9091	36.88%	0.6172
RUSSIA	Fractal wave 1	HP	2.020	-1.020	1.272	0.966	100.00%	0.9922
	Fractal wave 2		1.194	-0.252			91.04%	0.9844
	Fractal wave 3		0.034	-0.116	-0.344	0.847	38.89%	0.9922
	Fractal wave 4		0.300	0.312	0.200	-0.792	26.60%	0.6172
CHINA	Fractal wave 1	HP	2.0105	-1.0105	1.3806	0.9949	100.00%	0.8906
	Fractal wave 2		0.6116	0.2282			**64.32%**	0.8047
	Fractal wave 3		-0.0037	-1.0674	-0.2107	0.9416	55.89%	0.0046
	Fractal wave 4		-0.1124	0.9494	0.5874	-0.3933	78.59%	0.4844
BRAZIL	Fractal wave 1	HP	1.9869	-0.9865	1.2850	0.9950	100.00%	0.8828
	Fractal wave 2		1.7403	-0.7693			98.73%	0.8750
	Fractal wave 3		-0.9369	-0.4817	0.5558	0.4104	25.06%	0.0781
	Fractal wave 4		0.9817	-0.0841	-1.1113	0.2875	18.07%	0.9844
US	Fractal wave 1	HP	2.0488	-1.0494	0.8139	0.9616	100.00%	0.8984
	Fractal wave 2		1.5436	-0.5982			95.07%	0.8750
	Fractal wave 3		-0.8097	-0.9103	1.5328	0.9586	80.73%	0.1016
	Fractal wave 4		0.5903	0.2183	-0.8440	0.9120	65.00%	0.8047

The efficient estimates of the Spectral density function of one random variable could be produced by using parametric approach or using on ARMA parameters .What is important for further analysis is the fact that every pure sine wave can be described by using AR(2) model [16]:

$y_t=2*\cos(2\pi f_m + \omega\mu)y_{t-1} - y_{t-2},$

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where f_m stands for frequency and ω_m stands for the phase .In reality the most of sine waves are damped sine waves which have second AR parameter slightly different from one (see equations 8 and 9).

Spectral density function of AR(2) process can be calculated using the following formula

$$S(w)=\frac{\sigma_e 2}{A^{(e-i\omega)}A^{(e+i\omega)}}$$

The results are presented in the figure for US GDP, the third fractal wave.1.It has a peak when frequency is 0.1016 (π).

All the other frequencies which maximize spectral densities for the corresponding wave fractals are presented in the last column of the table 3.

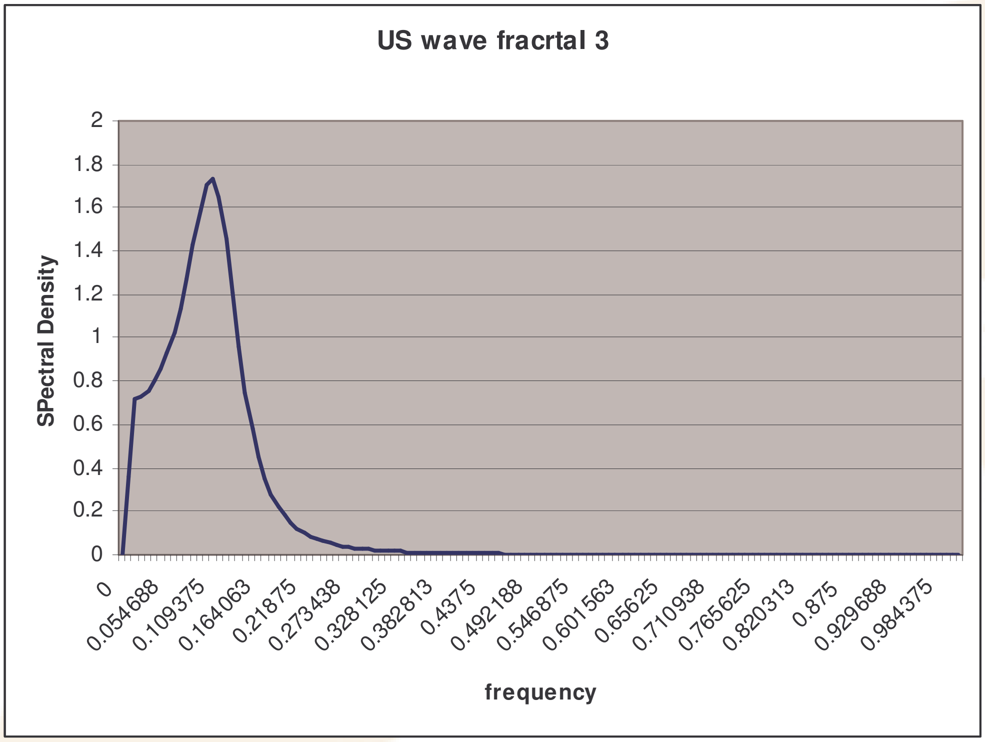

Figure 1: Third US fractal sine wave

The obtained results clearly show: that GDP cycles exist in BRIC countries as well as in US ; that cycles are fractal in Elliot's sense and that they can be modeled by a sequence of filters which extract signals with progressive frequencies.

Conclusion

Spectral Analysis is assuming a new importance since wide variety of methods, like AT&CF method for the currency rate fluctuations and the methods which produce countercyclical policies on GDP cycles , can be treated as problems in Spectral analysis . Those countercyclical policies can be of paramount importance .

In this paper it was demonstrated The real business cycle theory is more subtle than it appears at first blush. It was proved that H&P trend is smooth and produces residuals

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which can be described using a sequence of several AR filters, each being used for the extraction of sine waves. This contradicts results described by (Ivanov ,2005).
It was also shown that GDP cycles exist; that cycles are fractal in Elliot's sense. The results show evidence that GDP shocks can be modeled and forecasted using multi step Spectral analysis which determines the carrier frequency of the wave fractals for all BRIC countries as well as US. Amplitudes and phases of the dumped sine waves can be further calculated using the procedure carefully described in (Box& Jenkins 1995). The results suggest that the theory's implications for existing countercyclical policies remain a matter of further debate .

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