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Distressed property valuation and optimization of loan restructure terms

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Abstract

A homeowner currently residing in a home worth less than the mortgage frequently decides to walk away. This contributes to reduced market values of nonperforming loans and an illiquid real estate market with uncertain housing prices. The absence of loan restructuring plans beneficial to both homeowners and lienholders has perpetuated the problem. This study proposes a constrained optimization model that provides distressed property valuation and optimal loan restructuring terms. We are able to obtain these results by acknowledging homeowners have a real option and incorporating the value of that option in the maximization problem.

Key words: real estate, restructure, option, market value, real option

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1. Introduction

Option theory has been applied to real estate investment, abandonment, and timing decisions for some time now (Yavas and Sirmans, 2005). However, prior real estate applications of option theory are from the perspective of developers or financial institutions. Our study is the first to apply option theory to real estate from a household perspective. While prior studies have used option theory to determine *mortgage* values (Kau and Keenan, 1995) this study extracts *home* values from the option pricing framework. We then use the estimated market value in a constrained maximization problem.

The market value of non-performing loans averaged 23% of book value in 2009 (see Fig. 1). During that same year the average market value for performing loans was two and a half times better at 58% of book value. What if a loan were to be restructured such that it becomes a performing loan? Would that restructured loan, even with a possibly lower book value, have a higher market value than the original non-performing loan? If yes, what new loan terms are in the best interest of the homeowner and the lienholder? Also, would such an optimization problem provide insight to the current home value estimate? This study takes several steps toward answering these important questions.

[Figure 1 about here.]

The precipitous drop in home loan market values coupled with increased reserve requirements has necessitated loan restructuring. However lenders are often unwilling to restructure loans. We suggest one reason lenders are apprehensive is that they are unaware of the related issues of current home valuation and optimum restructuring terms. This scenario has been referred to as "A lender's dilemma" by Berger et al. (2009). Our study proposes a constrained optimization model that provides optimal loan restructuring terms and distressed property valuation. We are able to obtain these results by observing that homeowners have a real option and incorporating the value of that option in the maximization problem.

To illustrate the potential benefits to lienholders and homeowners consider the 2009 market to book values for performing and non-performing loans. Given market-to-book values of 23% and 58% for non-performing and performing loans, respectively, the restructured loan market value will exceed the non performing loan market value as long as the restructured principal balance is greater than 40% of the original principal balance. For example, assume an original loan balance

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of \$100,000 and this loan is now non-performing. Further assume that after restructuring the loan it is performing but has a lower principal balance of \$60,000. The market values before and after restructuring are:

$$MV_{\text{before}} = MV_{\text{nonperf}} = 0.23(100,000) = 23,000$$
$$MV_{\text{after}} = MV_{\text{restructured}} = 0.58(60,000) = 34,800$$

The lienholder can increase the market value of the non-performing loans by restructuring into performing loans. Of course there are many other choice variables for the lienholder such as the loan rate and term. The trio of *loan reduction percentage*, *loan rate*, and *loan term* are used as the lienholder's choice variables in our model that follows.

2. Model

The lender chooses the loan balance reduction amount a, the new loan interest rate r and the new loan term T to maximize the current value of the newly-formed performing loan subject to several constraints.

$$\max_{a,r,T} \quad b_{\text{p}}(aK_0) \quad \text{subject to} \quad \begin{cases} c[a,r,T] \geq 0 & (C1) \\ b_p(aK_0) \geq b_{np}K_0 & (C2) \\ PMT_p[a,r,T] \leq \delta PMT_{np} & (C3) \\ r \geq r_f + mrp & (C4) \\ 0 < T \leq 30 & (C5) \end{cases} \tag{1}$$

where b_p is the proportion of performing loan balance that translates into market value, $a < 1$ is the portion of the original non performing loan balance K_0 retained in the new loan, (aK_0) represents the restructured loan balance, $c[\cdot]$ is the value of the real option on the home, b_{np} is the proportion of the non-performing loan balance that translates into market value, $PMT_p[\cdot]$ is the monthly mortgage payment of the new performing loan, $PMT_{np}[\cdot]$ is the monthly payment of the old non performing loan, $\delta \leq 1$ is the amount of payment reduction, and r_f is the risk free rate. The endogenous variables are the lender's choice variables (a, r, T), i.e., the new loan terms. The exogenous variables are the current home price S_0, risk free rate r_f, mortgage loan risk premium mrp, volatility σ, time commitment to remain in the home t, market-to book value ratios (b_p, b_{np}), and payment reduction δ.

2.1. Assumptions

To make our model tractable several simplifying assumptions are made. These assumptions are intentionally restrictive. We do not intend for this model to apply to all non-performing loans on the market. However, loans that meet the criteria identified in the following assumptions do represent a subset of all non-performing loans.

Assumption 1: The loan non performing loan represents a scenario where the real option value on the home is zero. The "underwater" loan, i.e., a situation when the loan balance exceeds the property value, is such that the real option on the home is worthless. Admittedly there are other possible reasons for a loan to be non-performing. The homeowner may have experienced unemployment or under-employment recently. We accommodate for that via a payment constraint in the model. Another reason may be the homeowner is an investor who has walked away from a property in which mortgage payments exceed rental income. Again, we accommodate by incorporating a payment constraint.

Assumption 2: The loan will become performing after restructuring. After restructuring according to the terms of our model the loan will have satisfied payment constraints as well as reestablishing a positive real option value on the home. Assuredly not all home loans can be restructured. However, we consider successful convergence to a solution, subject to several constraints, an indication of applicability. If the model converges to a solution given the inputs we consider that the loan is a restructure candidate. If the model does not converge and satisfy the constraints the loan is not a restructure candidate.

Assumption 3: The home price process follows a lognormal distribution. We make this assumption to apply the more tractable binomial option pricing valuation to our model. This is a standard assumption with option pricing made in Hull (2006) and implemented in Benninga and Wiener (1997a,b).

Assumption 4: The homeowner will agree to remain in the home or make payments for t years. We acknowledge that the same agreement was made in the now non-performing original loan. However, since the output of our model is a restructured loan beneficial to both the homeowner and the lienholder we consider t, which is distinct from the loan term T, a required commitment from the homeowner for restructuring to occur.

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Additional simplifying assumptions. To make the model more tractable we further assume the market-to-book-value of non-performing loans and performing loans remains constant. In line with option pricing practice, we also assume that price volatility and the risk free rate are stationary, there are no transaction costs or taxes, and the stock (home) does not pay a dividend. Finally, we assume the restructured loan is a fixed rate loan.

2.2. Constraints

Given the number and importance of the constraints we devote the following section to explaining the constraints in detail.

Constraint $C1$: The option value under the restructured loan must be greater than zero. This follows Assumption 1 in that a loan becomes non performing when the option value is zero. A performing loan has a nonzero and positive real option value to the homeowner. The real option value is computed using the Black-Scholes option pricing model:

$$c\left[a,r,T\right]=S_0N\left[d_1\right]-K_te^{-r_ft}N\left[d_2\right]\geq 0 \tag{2}$$

$$d_1=\frac{\ln\left[S_0/K_t\right]+\left(r_f+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \tag{3}$$

$$d_2=d_1-\sigma\sqrt{t} \tag{4}$$

where S_0 is the current market value of the home, t represents the time commitment of the homeowner (see Assumption 4), and K_t is the principal balance at time t represented by:

$$K_t=\left(aK_0\right)\frac{1-e^{-r(T-t)}}{e^{-rT}} \tag{5}$$

Constraint $C2$: The market value of the restructured performing loan must exceed the market value of the current non performing loan. This constraint is included to ensure that restructuring is beneficial to the lender. If this constraint were not in place, it could be possible to restructure into a performing loan whose market value is lower than the current non performing loan.

$$b_p\left(aK_0\right)-b_{np}K_0\geq 0 \tag{6}$$

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Constraint C3: The new payment must be proportionately less than the current payment. Payments are not being made on the current non performing loan. As such we constrain the problem by setting the new payment to a value proportionately less than the current payment:

$$PMT_p\left[a, r, T\right] - \delta PMT_{np} \leq 0 \tag{7}$$

The monthly payment is calculated using the standard continuous time formula:

$$PMT_p = \frac{(aK_0) \times {}^{r}\!/_{12}}{1 - e^{-rT}} \tag{8}$$

$$PMT_{np} = \frac{A \times {}^{r_{orig}}\!/_{12}}{1 - e^{-r_{orig}T_{orig}}} \tag{9}$$

where A is the original non performing loan amount, r_{orig} is the original non performing loan interest rate, and T_{orig} is the original non performing loan term.

Constraint C4: The new loan rate must be greater than or equal to the risk free rate plus the risk premium for performing loans. Unlike the borrowing assumption of the option valuation model the new loan rate must be above the risk free rate in our model[1].

$$r - r_f - mrp \geq 0 \tag{10}$$

where mrp is the risk premium for performing 30 year loans.

Constraint C5: The maximum new loan term is 30 years. Given the finite lives of homeowners and customary industry practice we restrict the maximum new loan term to 30 years.

$$0 < T \leq 30 \tag{11}$$

2.3. Implementation

We propose a two step process to the maximization problem. First we establish the value of the home S_0 by setting the value of the current real option on the home to zero; then we use that home value in our constrained optimization model.

[1]We see no reason to believe a lender can restructure a loan into a rate lower than the risk free rate.

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2.4. Determining the current home value

The current home value is obtained by solving the real option on the home for the current value of the home:

$$S_0 N[d_1] - K_t e^{-r_f t} N[d_2] = \gamma S_0 + (K_0 - S_0) \tag{12}$$

where d_1 and d_2 are defined by Eq. (3) and Eq. (4), K_t is the strike price or future value of the loan if the home were purchased at time 0, and γ is the down payment percentage. Note that K_t represent the future principal balance of a loan if the home were purchased at time 0, for price S_0, at interest rate r, and with down payment requirement γ. As such, K_t is computed as

$$K_t = (1-\gamma) S_0 \frac{1 - e^{-r(T-t)}}{e^{-rT}} \tag{13}$$

The right hand side of Eq. (12) requires further explanation. If the underlying asset of the non performing loan (the home) were to be purchased to day the purchaser would also be responsible for the liability associated with the home K_0. Thus, the cost of the real option on the home is the down payment γS_0 plus the amount the current non performing loan balance exceeds the current market value. This ensures an initial loan to value ratio of $1-\gamma$. In other words, the real option holder will have an initial loan balance of $(1-\gamma) S_0$ after paying $\gamma S_0 + (K_0 - S_0)$.

Unfortunately, obtaining an explicit closed form solution of Eq. (12) using either the Black-Scholes model or the binomial option pricing model is difficult[2]. However, numerical methods can be used to solve Eq. (12) for S_0.

2.5. Constrained optimization model

As with the determination of the current home value, obtaining an explicit closed-form solution to the constrained optimization problem is difficult. Again numerical optimization methods can be called upon to solve the constrained optimization problem in Eq. (1). Specifically, after defining the exogenous variables S_0, r_f, mrp, σ, t, b_p, b_{np}, and δ, numerical methods are employed to obtain the optimized values of lender choice variables (a^*, r^*, T^*).

[2]We won't say impossible. But we, and Mathematica, have been unable to do so thus far.

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3. Numerical illustration

We begin by defining our constant exogenous variables. Table 1 summarizes the parametrisation of our numerical illustration.

[Table 1 about here.]

The nationwide average rate for 30 year performing loans according to was 4.5%. Given the risk free rate and market rate for 30 year loans we set the mortgage risk premium $mrp = 0.0450 - 0.0375 = 0.0075$. Fig. 2 depicts the Case-Shiller composite home price index returns from 1998 to 2010. The annual volatility of the Case-Shiller Home Price Index over that time period is 9.26% and we set $\sigma = 0.0926$.

[Figure 2 about here.]

Assume a currently non performing loan with balance $K_0 = 129,375$, an original loan balance of $A = 140,000$, and original interest rate of $r_{orig} = 6.375\%$, and original loan term of $T_{orig} = 30$. Given the parametrisation of Table 1 and the assumed current non performing loan information our model in Eq. (12) produces an estimate of $S_0 = 102,535$. Given that the assumed non performing loan values were from a home we are familiar with we decided to compare the output of our model with web-based estimates. As of November 30, 2010 ZILLOW.COM estimates the value as 109,000 and EAPPRAISAL.COM estimates the value as 114,061. These results are summarized in Table 2 Panel A. We conclude the estimate produced by our model is on par with current market conditions.

[Table 2 about here.]

We also perform a sensitivity analysis to investigate factors that impact market values. The results of the sensitivity analysis are presented in Fig. 3. Several observations are of note. First, the home value is most sensitive to the anticipated time in home and the home value decreases with time in the home. This result is counter intuitive and we believe emphasizes the importance of closing costs. Second, and also counter intuitive, is the positive relationship between mortgage rates and home value. Third, spot market values appear to be insensitive to historical volatility.

We believe our last observation is of particular note. The home value increases as down payment requirements increase. Higher down payment requirements benefit both the lender via lower risk loans and the borrower via lower payment and higher property value. Ironically, during the housing

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boom down payment requirements were often zero or even less than zero. Such practices ultimately led to the housing bubble and burst. In a sense, our model predicts lower home values when down payment requirements are loosened.

[Figure 3 about here.]

Armed with an arguably accurate and reliable home value estimate we proceed to determining optimum loan restructuring terms. We vary the payment reduction percentage δ from 0.5 to 1 to assess the efficacy of loan restructuring. The results of the maximization problem Eq. (1) are presented in Table 2 Panel B. Our model shows that all scenarios where the loan payment is reduced by 50% or more produce a new performing loan that is more valuable than the old non performing loan. To illustrate consider the fact that the original non performing home loan has a market value of 62,998. When the original loan principal balance is reduced 20.44% $(1 - a^*)$ the resulting mortgage payment is reduced 40% and the market value of the new performing loan increases to 84,056. This is a benefit to the bank (+21,058 increase in market value of loan) and the homeowner (20.44% reduction in principal balance and 40% reduction in monthly payment).

4. Conclusion

We have presented a method to establish the market value of distressed property and a model to determine optimum loan restructuring terms. The distressed property home valuation approach utilizes the fact that a home loan is a real option on a home. The Black-Scholes option pricing formula was used in conjunction with numerical methods to arrive at a market value estimate very close to that of popular web-based tools. Sensitivity analysis revealed the importance of elevated down payment requirements and the benefits thereof: higher market values and lower risk loans. We used our home value estimate and varied the desired payment reduction to produce optimum loan restructuring terms. We found that the model does produce results beneficial to both the lienholder and homeowner. Specifically, we presented scenarios where a principal and interest rate reduction can increase the market value of the loan and reduce the monthly payment for the homeowner to an affordable level.

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Table 1: Parameters, values, and sources

Symbol	Description	Value	Source
r_f	risk free rate	3.75%	30 year U.S. T-bond average in September 2010
mrp	mortgage risk premium	0.0075%	Bankrate.com, September 2010
σ	annual home price volatility	9.26%	S&P/Case-Shiller
b_{np}	long run average non performing loan market to book ratio	0.4846	FDIC historical home loan sales data
b_p	long run average performing loan market to book ratio	0.8127	FDIC historical home loan sales data
t	average length of home ownership	6 years	Google search, National Association of Realtors
γ	average down payment requirement	10%	mortgageqna.com

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Table 2: Model outputs

Home value estimates and loan restructuring model outputs. In Panel A we present estimates of the distressed property home value from our model and two popular websites. In Panel B we present the outputs from our optimization model:

$$\max_{a,r,T} \quad b_{\mathrm{p}}\left(aK_0\right) \quad \text{subject to} \quad \begin{cases} c\left[a,r,T\right] \geq 0 & (C1) \\ b_p\left(aK_0\right) \geq b_{np}K_0 & (C2) \\ PMT_p\left[a,r,T\right] \leq \delta PMT_{np} & (C3) \\ r \geq r_f + mrp & (C4) \\ 0 < T \leq 30 & (C5) \end{cases}$$

where b_p is the proportion of performing loan balance that translates into market value, $a < 1$ is the portion of the original non performing loan balance K_0 retained in the new loan, (aK_0) represents the restructured loan balance, $c\left[\cdot\right]$ is the value of the real option on the home, b_{np} is the proportion of the non-performing loan balance that translates into market value, $PMT_p\left[\cdot\right]$ is the monthly mortgage payment of the new performing loan, $PMT_{np}\left[\cdot\right]$ is the monthly payment of the old non performing loan, $\delta \leq 1$ is the amount of payment reduction, and r_f is the risk free rate. The endogenous variables are the lender's choice variables (a, r, T), i.e., the new loan terms. Also note that the market value of the new loan $MV_{new} = b_p\left(a^*K_0\right)$ while the market value of the original non performing loan is $MV_{old} = b_{np}K_0 = 62,998$.

Panel A: Home value estimate

Our model, Eq. 12	Zillow.com	eappraisal.com
102,535	109,000	114,061

Panel B: Optimization model output

δ^*	a^*	r^*	T^*	MV_{new}	$c\left[a^*, r^*, T^*\right]$
0.5	0.6630	4.500%	30	70,046	41,804
0.6	0.7956	4.500%	30	84,056	30,062
0.7	0.9282	4.500%	30	98,065	19,729
0.8	1.0000	4.516%	30	105,651	15,066
0.9	1.0000	4.615%	30	105,651	14,958
1.0	1.0000	4.613%	29.7	105,651	15,083

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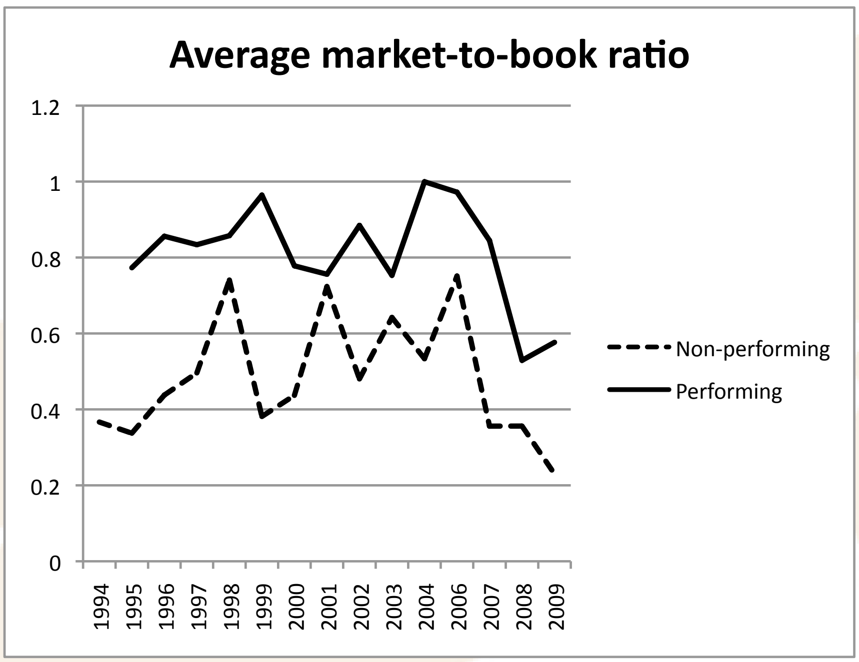

Figure 1: Average market-to-book ratio for performing and non-performing loans from 1994 to 2009. Source: FDIC historical loan sales data (http://www.fdic.gov/buying/historical/index.html).

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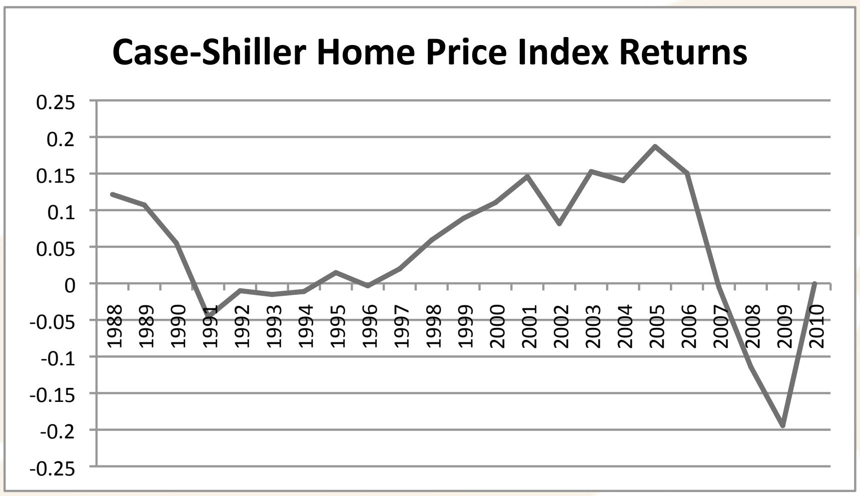

Figure 2: Composite housing price index returns from January 1988 to June 2010. Source: S&P/Case-Shiller Home Price Indices.

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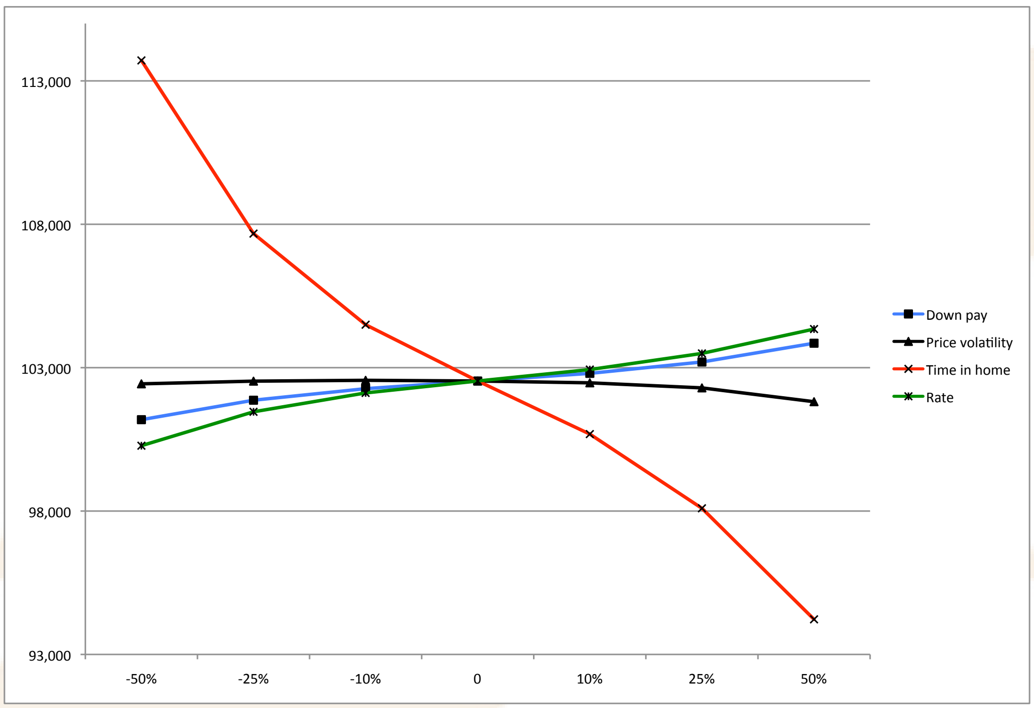

Figure 3: Home value sensitivity to changes in model inputs