Stock Market Volatility Forecasting using Higher Order Cumulants –Evidence from the International Stock Markets

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Abstract: The aim of this paper is to propose the HOC- GARCH stock market volatility model and apply it to the cases when returns have a non-Gaussian distribution or when distribution of errors is unknown. Higher Order Cumulants (HOC) are used not only as a means for non-Gaussian analysis but also to estimate the parameters of the ARMA version of the HOC-GARCH volatility model. The first step of the empirical analysis includes the lagged multiple regression model for the daily Swiss stock market index to include DJIA, SP500, NASDAQ ,DAX, FTSE100, Nikkey225, BSE as well as its own trading volume as its determinants. In the second step, SMI squared residuals are found to be non-Gaussian and to exhibit volatility clustering. They are described by using a HOC-GARCH (4,4) model and by using a new parameter estimation method based on fourth order cumulants. The results show that parameter estimation based on the third- and fourth-order cumulants successfully captures non-Gaussian features of the stock market returns and thus improves the stock market volatility forecasting methodology, without limiting the application to the a priori known r distributions. The time horizon includes Oct 4, 2007 and Oct.12, 2010. All data are taken from Bloomberg.

Key Words: Volatility Forecasting, Stock Market Returns, Higher Order Cumulants, GARCH model, ARMA Model, Swss Market Index .

JEL Classification Numbers: G15, G17

Introduction

Amid the today's debate on stock market volatility forecasting, the choice of the best model or the best estimation method with the goal to extract information from available stock market data remains at the forefront of the discussion.

While over the last two decades most researchers and practitioners have pursued this goal through the empirical use of the well known Autoregressive Conditional Heteroskedastic (ARCH) model or its generalized form, known as the GARCH model, the evaluation of the forecasting properties of the model itself shift the interest towards GRACH model optimization.

As it was pointed out by Nelson (1991), there are at least three drawbacks of the GARCH model. Firstly, the model assumes that only the magnitude and the positivity or negativity of unanticipated excess returns determines the feature of volatility. Black (1976) noted the tendency for negative Innovations to generate greater volatility in future periods compared with positive innovations of the same magnitude, a phenomenon that he refers to as the "leverage effect". The second drawback results from the limitation on the model parameters which are imposed to insure that the estimated variance remain non negative (Engle, Lillian and Robins 1987) .The third drawback of the GARCH model concerns the interpretation of the "persistence" of the shocks to the conditional variance.

Ideally, a good volatility model should be able to capture and reproduce most, if not all, of these drawbacks and allow the same degree of simplicity and flexibility in representing the conditional variance as ARIMA models have allowed in representing the conditional mean.

An important contribution in improving GARCH is given by Nelson (1991), who proposed an exponential GARCH or EGARCH (1,1) model that, in addition, contains the parameter γ. It is expected that $\gamma < 0$, "good news", generates less volatility than "bad news", where γ reflects the

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leverage effect. He also originally assumed that the errors follow t or Generalized Error Distribution (GED). It is the only example of the GARCH model with non-Gaussian returns developed so far. The parameter estimation method was accomplished by using a maximum likelihood method and the subroutine DUMING, available in the IMSL statistical library. Nonetheless, the model has the important drawback that the forecast of volatility requires a distribution assumption or its numerical simulation (see Engle, 1995, pg xiii).
The GJR model is proposed by Glosten, Jagannathan & Runkle (1993). The variance equation in a GJR (1,1) model contains an indicator (I) as a dummy variable that takes the value 1 if $\varepsilon<0$, and zero otherwise. This model suffers from the second drawback of the standard GARCH model. Ling & McAleer (2002) established the regularity condition for the existence of the second moment of the GJR (1,1) model.

Another asymmetric variant of the GARCH model is the threshold GARCH (TGARCH) model proposed by Zakoïan (1994). It is similar to the GJR, but the model uses the conditional standard deviation instead of the conditional variance.

As pointed out by Engle (1995), despite the success of the standard GARCH (1,1) the model and its modifications in describing the dynamics of conditional volatility in financial markets (particularly in the short run), its implications for long run volatilities are restrictive, in the sense that this class of models imply a constant expected volatility in the long run (i.e., the long run volatility forecast is constant). The SPLINE-GRACH model which is flexible enough to generate an expected volatility that captures the long run patterns observed in the data was proposed by Engle & Rangel (1995).
To accomplish the goal, they modified the standard GARCH (1,1) model by introducing a trend in the volatility process of returns. Specifically, this trend is modeled non-parametrically using an exponential quadratic spline, which generates a smooth curve describing the long run volatility component based exclusively on macroeconomic data evidence.

An investigation of the relative performance of GARCH models versus simple rules in forecasting volatility is done by Silvey (2009). While numerous studies have compared the forecasting abilities of the historical variance and GARCH models, no clear winner has emerged. In a thorough review of 93 such studies, Poon and Granger (2003) report that 22 find that historical volatility forecasts future volatility better out-of-sample, while 17 studies find that GARCH models forecast better. Brooks (1998) used DJ composite daily data to test in- and out-of-sample forecasts obtained with GARCH, EGARCH, GRJ and HS (historical volatility) models. The R^2 achieved was around 25% for each of the models. Ederington and Guan (2004) used GARCH, EGARCH, HIS and AGARCH to evaluate forecasts of the DJ and S&P daily volatilities .They found that absolute returns perform better than squared returns, but without achieving any statistical difference between the performances of the forecasting models .

Summing up , there are many variants of ARCH-GARCH models which are developed to improve the "out of sample" volatility forecasting performance. They have many strong supporters, who believe that those models are currently the best obtainable models. However, most of the studies found no clear-cut results in improving forecasting performances of the class of GARCH models (Poon & Granger 2003 and Carrol & Kearvey 2009). Indeed, there are studies which confirm a very low coefficient of determination produced by GARCH models. For instance, Anderson and Bolleslev (1988) showed that R^2 for a GARCH (1,1) model tends to $1/\kappa$, where κ stands for the kurtosis of the distribution of stock returns. This means that the highest R^2 for Gaussian returns achievable by GARCH models is bounded from above by 1/3. For the stock market returns, which have a non-Gaussian distribution, a kurtosis is usually higher than three, which causes that volatility forecast performance to be worse.

The nature of this GARCH controversy does not seem to focus upon the model structure but rather upon the distribution of the returns for which second order moments do not represent a sufficient statistic for the parameter estimation.

The aim of this paper is to develop a HOC-GARCH model by suggesting a new parameter estimation method for improving the GARCH volatility forecasting for non-Gaussian returns which is to be applied when the distribution of the errors is not known a priori. Thus a new approach is aimed to solve a drawback of the EGARCH model. A Parameter estimation method using higher order cumulants has not been investigated in the financial literature so far, and the present paper fills that gap.

The proposed method is tested by using daily closing prices of the SMI, DJIA, SP500, DAX, FTSE100, NASDAQ and BSE indexes, for the period between Oct. 4, 2007 and Oct.12, 2010.
The paper is organized as follows: The second section presents the GARCH –ARMA model .The third section presents a new cumulant based parameter estimation method. The fourth section presents the data description, HOC, obtained volatility model, input and output fourth order cummulants. The final section presents the conclusions and suggestions for further research.

2. The problem and the model

Statistical properties of stock market returns, common across a wide range of developed stock markets and time periods, are called stylized facts. Stylized statistical properties of asset returns of developed markets are analyzed empirically and subsequently summarized by Cont (2001). They include the following findings:
The autocorrelations of asset returns are often insignificant, except for high frequency data ($f \simeq 20$ minutes or less); b) Heavy tails, with a finite tail index, which is higher than two and lower than five for most data sets studied; c) Gain/loss asymmetry: one observes large drawdown in stock prices and stock index values but not equally large upward movements; d) Aggregate Gaussianity according to the central limit theorem; e) Volatility clustering, namely, different measures of volatility display a positive autocorrelation over several days, which quantifies the fact that high-volatility events tend to cluster in time; f) Conditional heavy tails even after correcting the returns for the volatility clustering ;g) Slow decay of autocorrelation in absolute or squared returns, i.e. non-stationarity; h) Leverage effect; most measures of the volatility of an asset are negatively correlated with the returns of that asset; i) Volume/volatility correlation i.e., the trading volume is correlated with all measures of volatility

The fact that market returns are often characterized by volatility clustering, which means that periods of a high volatility are followed by periods of a high volatility and periods of a low volatility are followed by periods of a low volatility, implies that the past volatility could be used as a predictor of the volatility in the next periods. As an indication of volatility clustering, squared returns often have significant autocorrelations and consequently can be modeled by using the well known GARCH model.

Let e_t denote a discrete time stationary stochastic process. The GARCH (p, q) (Generalized Autoregressive Moving Average Conditional Heteroskedasticity) process is given by the following set of equations (Bollerslev, 1986,42-56):

$$r_t = \log(p_t) - \log(p_{t-1}) \quad (1)$$

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$r_t = x_{(k)} g_{(k)} + e_t$ (2)

$e_t = v_t \sqrt{h_t}$

$e_t / _{t-1} \approx N(0,h_t)$, (3)

$$h_t = a_0 + \sum_{i=1}^{p} \alpha_i e^2_{t-i} + \sum_{j=1}^{q} \beta_j h_{t-j} \quad (4)$$

in which p_t represents stock prices, r_t returns, $x_{(k)}$ vector of explanatory variables, $g_{(k)}$ vector of multiple regression parameters, h_t conditional volatility, α_i autoregressive and β_j moving average parameters as related to squared stock market index residuals.
An equivalent representation of the GARCH (p, q) model is given by:

$$e_t^2 = \alpha_0 + \sum_{i=1}^{P} \alpha_i e^2_{t-i} + \nu t + \sum_{j=1}^{q} \beta_j \nu_{t-j} \quad (5)$$

where $\nu t = e_t^2 - h_t$ and where, by definition, it has the characteristics of (i. i. d) white noise .
In other words, the GARCH (p, q) volatility model is an Autoregressive Moving Average (ARMA) model in e_t^2 driven by white noise ν_t which is not necessarily Gaussian.
In the case of stock market returns, driving noise is not, most commonly, Gaussian and subsequently the second order moment of the associated probability density distribution are not a “sufficient statistics” for the ARMA parameter estimation. In fact, it is well known that for non-Gaussian process, a higher order moment exists and is different from zero. The hypothesis in this article is that higher order moments contain the information necessary to capture heavy tails and volatility clustering.

HOC estimation method

Realizing, as early as in 1899, that the normal distribution was unsatisfactory for describing economic and demographic data, Danish statisticians Thiele and Gram, proposed to multiply the normal density by a power series and determine the coefficients by least squares, which lead to Gram Gram-Chariler series and a new system of skewed distributions f(x):
$f(x) = \exp[-(\kappa 1-\mu) +(\kappa 2-\sigma 2)/2! -\kappa 3/3! +...](2\pi\sigma^2)^{-1/2} \exp[(-(x-\mu)^2/\sigma^2]$, where □m is cummulant of order m. Moments of arbitrary order may be calculated via a distribution's *moment- generating function* $M_x(t) = E\{e^{xt}\} = \int e^{xt} f_x(x)dx$. The n-th order moment of a signal is generated by differentiating $M_x(t)$ *n* times, and setting t =0. Thus, the mean of a signal may be calculated as $\mu_x = E\{X\} = M_x'(0)$, where the single prime mark indicates the first differential of $M_x(t)$. Moments of higher order may be generated similarly.
The normalized n^{th} central moment of x, or standardized moment, is the n^{th} central moment divided by σ^n: $E((x - \mu)^n)/\sigma^n$. The normalized central moments are dimensionless quantities.
The third central moment is a measure of the symmetry of the distribution, which is called the skewness and is often marked as γ. A distribution that is skewed to the left (the tail of the distribution is heavier on the left) will have a negative skewness. A distribution that is skewed to the right (the tail of the distribution is heavier on the right), will have a positive skewness.
The fourth central moment is a measure of whether the distribution is tall and skinny or short and squat, compared to the normal distribution of the same variance and the same mean. The fourth central moment of a normal distribution is $3\sigma^4$; for that reason its kurtosis is three.

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The cumulant generating function is $\kappa(t) = \ln \Psi(t)$, where the characteristic function is $\Psi(t) =[E \{ e^{ixt} \}]$. Taylor's expansion of this characteristic function was used by Bessel, Laplace and Poisson and discussed in Hald (2000), who pointed out that Theile was the first who proved the relationship between moments' derivatives and cumulants. Since all the cumulants are polynomials in the moments, so are the factorial moments.

Cumulants have certain important properties which are not found in moments. For instance, the cumulant for the sum of two independent signals is the sum of the cumulants, which does not hold for the moments. In addition, cumulants of the Gaussian process of the order higher than two are equal to zero.

During the last two decades, dynamic forms of higher order cumulants (lag≠0) have been used in many fields: e.g., signal data processing, adaptive filtering, harmonic retrieval biomedicine and image reconstruction. Unbelievable as it may seem, they have not been used in economics and finance. Nevertheless, there were trials in finance to use co-skeweness and co-kurtosis of order one to build Capital Asset Price model (Adesi 20003).

In the area of digital signal processing, Giannakis (1987) was the first to show that AR parameters of non-Gaussian ARMA signals can be calculated using the third- and fourth-order cumulants of the output time series given by:

$$C^3_x(\tau 1,\tau 2)= (\sum(x(t)x(t+\tau_1)x(t+\tau_2))/n, \qquad (6)$$

$$C^4(\tau 1,\tau 2,\tau 3,)= (\sum(x(t)x(t+\tau_1)x(t+\tau_2)\, x(t+\tau_3))/n -$$

$$-C^2_x(\tau 1)\, C_x(\tau 2-\tau 3) - C^2_x(\tau 2)\, C_x(\tau 3-\tau 1)-C^2_x(\tau 3)\, C_x(\tau 1-\tau 2), \qquad (7)$$

where n is a number of observations and where the second-order cumulant $C^2_x(\tau)$ is just the autocorrelation function of the time series x_t.

The zero lag cumulant of the order3 $C^3_x(0,0)$ normalized by $\sigma_x^{\ 3}$ is skewness γ^3_x; $C^4_x(0,0,0)$ normalized by $\sigma_x^{\ 4}$ is known as kurtosis $\gamma^4_{x.}$

A new method of the AR parameter estimation for non-Gaussian ARMA (p,q) processes is based on the modified Yule-Walker system where autocorrelations are replaced by third or fourth order cumulants (Gianninakis -1990):

$$\sum_{1=1}^{p} \alpha i\, C^3(k-i,k-l) \quad = - \quad C^3(k,k-l) \qquad k\geq l\geq q+1 \qquad (8)$$

$$\sum_{1=1}^{p} \alpha i\, C^4(k-i,k-l,\ k-m) \quad = - \quad C^4(k,k-l,\ k-m) \qquad k\geq l\geq\ m\geq q+1 \qquad (9)$$

The efficient MA parameter estimation can be performed by applying one of the proposed algorithms, for instance, q-slice algorithm (Swami 1989). Q–slice algorithm uses autoregressive residuals calculated after estimating the AR parameters or ARMA (10). Following up, the impulse response parameters ψ_I of the pure MA model of x_t model are then estimated using cumulants (11):

$$x_t=\sum_{0}^{\infty}\psi_j a_{t-j} \qquad i=1.2\ldots\infty \qquad (10)$$

$$\psi j = \frac{\sum_{1}^{p} \alpha i\, C^3(q-i,j)}{\sum^{p} \alpha i\, C^3(q-i,0)} \qquad j=1,2\ldots q \qquad (11)$$

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Or by using :

$$\psi j = \frac{\sum_{1}^{p} \alpha i\, C^4(q\text{-}i,j,0)}{\sum_{1}^{p} \alpha i\, C^4(q\text{-}i,0,0)} \quad j=1,2\ldots q \qquad (12)$$

The MA parameters of the ARMA model are obtained by means of the well known relationship:

$$\beta_j = \sum_{0}^{p} \alpha i\, \psi_{(j\text{-}i)} \quad j=1,2\ldots q \qquad (13)$$

The cumulants based ARMA estimates are shown to be asymptotically optimal by Friendler B. and Porat B. (1989).

3. Empirical results

The intention of this study is to use the stylized facts described above to develop a HOC-GARCH model for the volatility and apply it for the case of seven international stock markets returns.

Initially, the well known fact of the stock market co-movements is explored with the aim to remove the dynamical trend. Hence, the analysis consists of two steps. The fist step is aimed to find the best dynamic regression model in which the Stock index return's variance is explained by the international stock market indexes which Granger cause its change. After finding the regression model, whitened residuals e_i are calculated and squared. The squared residuals are than used to produce the HOC-GARCH model for the implied volatility of the returns.

The empirical analysis is based on daily quotations of indexes during the period from Oct.4, 2007 to Oct.12, 2010, taken from Blumberg. The first part of the analysis is done by Eview 5.1 software while the second part, which is related to the estimation using higher order cumulants, is done using MATLAB, its existing toolboxes and using subroutines - M files written by the author of this article.

The statistical description of the returns of these indexes is given in Table 1. The skewness and kurthosis factors, which are given in the table, show that all the variables are non-Gaussian (according to the skewness, kurtosis and the Jarque-Bera test for normality). The results of the Granger causality test for the returns and for the lag, l==1, are presented in Table 2. The test shows that returns of all the selected international stock market returns r^{DAX30}, $r^{FTSE}100$, r^{DJIA} and r^{SP500} do Granger cause SMI returns.

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Table 1: Statistical description of Stock Market Returns

	RBSE	RDAX	RDJI	RFTSE	RNAS	RSMI	RSP
Mean	0.069771	-0.00091	-0.01194	-0.01038	0.006283	-0.02527	-0.01752
Median	0.142909	0.08503	0.045429	0.00327	0.035221	0.029505	0.082768
Maximum	12.81051	10.62296	10.949	9.838667	9.239594	8.462643	10.67121
Minimum	-10.7595	-9.86026	-7.7927	-8.84835	-9.93517	-12.5336	-8.71188
Std. Dev.	2.466435	1.712902	1.701835	1.716069	1.829435	1.701808	1.776388
Skewness	-0.16844	-0.22935	0.294347	0.14743	-0.15261	-0.53534	0.011692
Kurtosis	6.009709	11.0191	10.18929	8.936601	8.18556	10.54147	9.732967
Jarque-Bera	283.1804	1991.942	1606.502	1090.819	833.106	1791.37	1399.668
Probability	0	0	0	0	0	0	0

Table 2: Granger causality Test results

Granger Causality Test Results		
Null Hypothesis:	Obs	**F-Statistic**
RSMI does not Granger Cause RDAX	760	5.04583
RDAX does not Granger Cause RSMI	760	0.04954
RSMI does not Granger Cause RBSE	760	0.87278
RBSE does not Granger Cause RSMI	760	0.11956
RSMI does not Granger Cause RDJI	760	8.26946
RDJI does not Granger Cause RSMI	760	2.76686
RSMI does not Granger Cause RFTSE	760	10.4573
RFTSE does not Granger Cause RSMI	760	2.35451
RSMI does not Granger Cause RNAS	760	16.6118
RNAS does not Granger Cause RSMI	760	6.35779
RSP does not Granger Cause RSMI	760	2.79157
RSMI does not Granger Cause RSP	760	8.87706

From Table 2 and equation (2) the most general dynamic model for the SMI returns can be deduced:

$$r^{SMI} = \phi_0 + \phi_1\, r_{t-1}^{SMI} + \phi_2 r_{t-1}^{SP} + \phi_3 r_{t-1}^{NAS} + \phi_4 r_{t-1}^{FTSU} + \phi_5 r_{t-1}^{DJI} + \phi_5^{SMIVOL} \quad (14)$$

With the purpose of digital whitening of the SMI returns, the best model is selected using the AIC criterion. The parameter estimates and standard deviations of those models are presented in Table 3.

Table 3: Dynamic regression model for the SMI index returns

SMI Returns	RSMI(-1)	RSP(-1)	RNAS(-1	RFTSU(-2)	RDJI(-1)	SMIVOL-1)
	$\phi1$	$\phi2$	$\phi3$	$\phi4$	$\phi5$	$\phi6$
	-0.10963 0.036031	-0.35552 0.019944	0.173316 0.061298	0.074296 0.035543	0.314187 0.017885	0.000317 0.036031

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The obtained results demonstrate that lagged returns of all six indexes are statistically significant. The American Stock Market indexes have higher influence than the European Stock Market indexes, with 1-day lagged SP returns being the most influential on the SMI returns. However, the coefficient of determination is rather small (about 3%).

With the objective to find the best GARCH model for squared returns' residuals, i.e. to find the volatility model for the SMI returns, the residuals are then calculated by using (12).

The statistical description of the squared SMI residuals, besides the skewness of 9.83 and kurtosis of 139.19, includes both the second and fourth order moments. The fourth-order cumulants are presented in Figure 2.

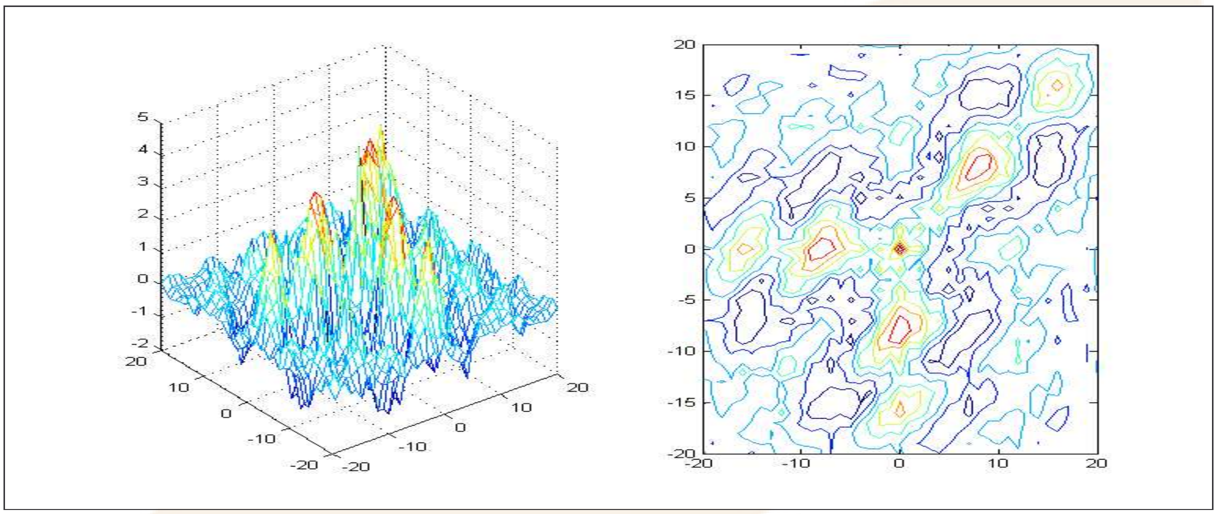

Figure 2: Fourth order cumulants of the squared SMI residuals.

Figure 2 shows that the fourth-order cumulants are different from zero. Consequently, the ARMA parameters are estimated using the fourth-order cumulants and the method described in Section Three. The best ARMA model parameters, based on the fourth order cumulants, for the SMI squared returns are found to be the following:

	AR(1)	AR(2)	AR(3)	AR(4)	MA(1)	MA(2)	MA(3)	MA(4)
Coefficient	-0.270608	0.100682	0.417762	0.573025	0.4113	0.231154	-0.36413	-0.36863
Std. Error	0.135599	0.042847	0.097438	0.093808	0.162061	0.095442	0.073164	0.099476
t-Statistic	-1.739144	2.349803	4.287444	6.108481	2.537932	2.421924	-4.97692	-3.70575

Or simply :

$$e_t^2 = -0.270\ e^2_{t-1} - 0.100\ e^2_{t-2} + 0.417\ e^2_{t-3} + 0.573\ e^2_{t-4} +$$
$$+ \nu t + 0.4113\ \nu_{t-1} + .232\ \nu_{t-2} - 0.364\ \nu_{t-2} - 0.368\ \nu_{t-2} \qquad (15)$$

This model has a property to capture information about heavy tails than the same type of the model based only on the second order moment. In fact, HOC-ARMA (4,4) residuals have cumulants which

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are presented in Figure 3.It can been seen that the cumulants are flat, which means that the information is extracted from squared residuals. The skewness and kurtosis of the ARMA residuals are found to be: 2.245 and 27.740 respectively.

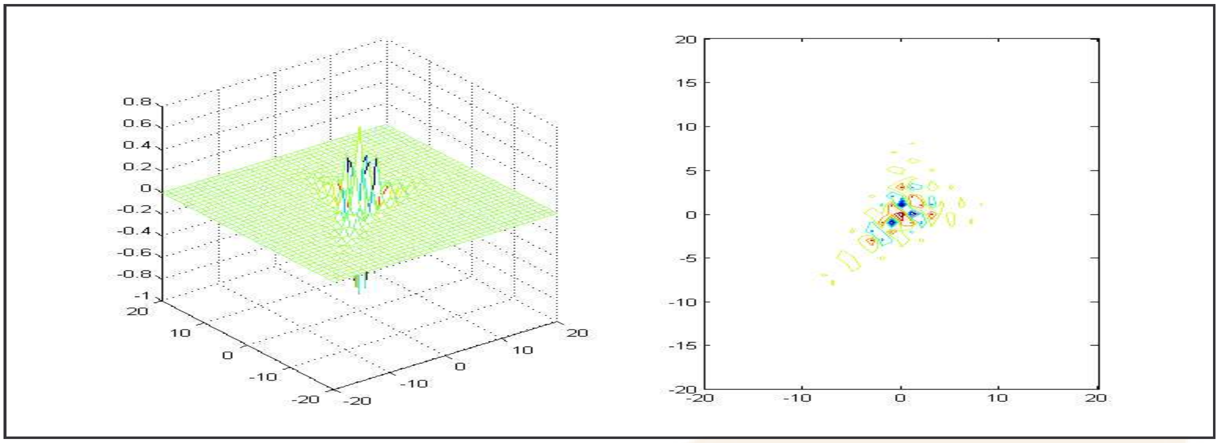

Figure 3: The fourth-order cumulants of the HOC-GARCH whitened residuals.

The GARCH equivalent of the ARMA model (15) has the form:

$$h_t = -0.270\, e^2_{t-1} - 0.100\, e^2_{t-2} + 0.417\, e^2_{t-3} + 0.573\, e^2_{t-4} + 0.4113\, h_{t-1} + .232\, h_{t-2} - 0.364\, h_{t-3} - 0.368\, h_{t-4} \quad (16)$$

Finally, the conditional volatility for the SMI index, which is calculated using this model, is presented in Figure 4.

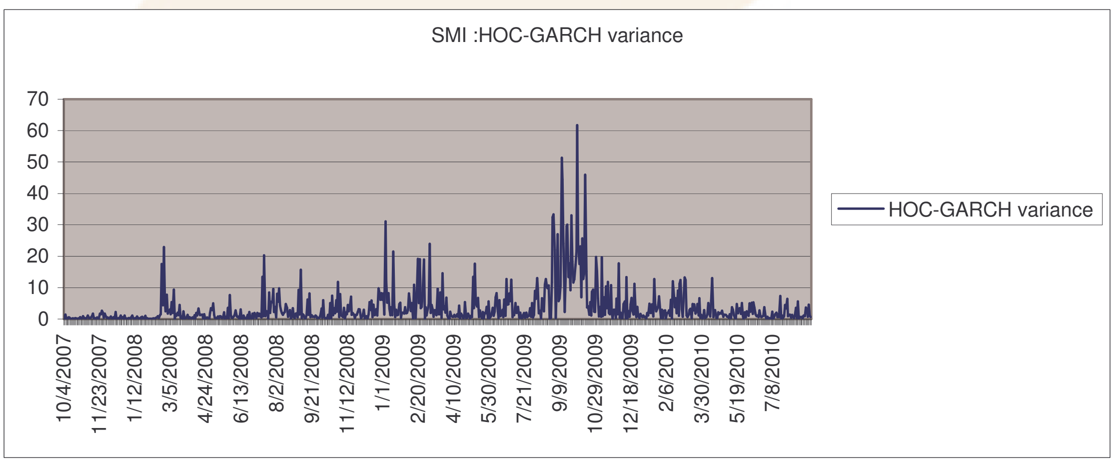

Figure 4: SMI conditional volatility

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4. Conclusion

Today the state of art of volatility forecasting improvement offers two paths, which lead either to a new analytical form of the GARCH class models or to the discovery of a new GARCH estimation method based on the sufficient statistics necessary to model non-Gaussian returns.

Ideally, a good volatility model should be able to capture and reproduce most, if not all of the generic GRACH drawbacks and allow the same degree of simplicity and flexibility in representing the conditional variance as ARIMA models have allowed in representing the conditional mean.

This paper has presented a new class of GARCH models, which do not suffer from some of the drawbacks of the GARCH, regarding stylized facts of asset returns such as asymmetry and heavy tails. A new class of models can be applied in the case when returns have distribution which is not a priori known, but is non-Gaussian. Indeed, the paper proposes to use a new estimation method, the HOC-ARMA parameter estimation method. Therefore the HOC estimation method is carefully described and applied for the first time to model the stock market volatility

The empirical analysis is based on daily SMI index data. The time horizon includes Oct 4, 2007 and Oct.12, 2010. The SMI returns are shown to be Granger caused by the DJIA, SP500, NASDAQ, DAX and FTSE100. Thus, the model building is accomplished in two steps. In the first step, a dynamic regression model is developed first in order to capture international stock market co-movements. Finally, the residuals, which result from such a model, characterized by almost no correlation, are then squared.

In the second step, which is essential for this paper, the squared residuals, which are proven to exhibit volatility clustering, are described by using the HOC-GARCH model. With the impetus of non-Gaussianity, GARCH parameter estimation is performed by using third- and fourth-order cumulants.

While the evidence from the International Stock Markets shows that the GARCH parameter estimation based on the higher order cumulants successfully captures non-Gaussian properties of the real stock market returns, the final research goal is yet to be accomplished: it remains to be investigated if squared residuals constitute a better proxy for the volatility forecast or HOC based historical volatility models perform more efficiently.

References:

Andersen T.G AND Bollerslev G.T. (1998) :"Answering the Skeptics : Yes , Standard Volatility Models Do provide Accurate forecasts ", Int.Econ.Revew, 39:4, 885-905.

Black, F. (1976). :Studies in stock price volatility changes". Proceedings of the 1976 Business Meeting of the Business and Economics Statistics Section, American Statistical Association, 177-181.

Bollerslev T.(1982),"Generalized Autoregressive Conditional Heteroskedasticity ", in ARCH Selected Readings ,ed by Engle R.,Oxford University Press,42-60

Cont Rama (2001):"Empirical properties of asset returns: Stylized Facts And Statistical Issues", Quantitative Finance, Vol.1, 223–236, http://www.proba.jussieu.fr/pageperso/ramacont/papers/empirical.pdf.

Carrol R. and Kearvey C(2009), " GARCH Modeling of stock market volatility " in Gregoriou G.(ed), Stock Market Volatility ,CRC Finance Series, CRC Presss, 71-90.

Ederington L. and Guan W(2004) ., "Forecasting Volatility ",SSRN Working paper series, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=165528

Engle R. (1995) , ARCH selected readings, Oxford university press.

Engle R. and Gonzalo J (1995) : The Spline GARCH Model for Unconditional Volatility and its Global Macroeconomic Causes, The Working Paper Series of the Czech National Bank(CNB) , http://www.cnb.cz/en/research/research_publications/cnb_wp/download/cnbwp_2005_13.pdf

Erjavec N. and Cota B.(2007),"Modeling Stock Market Volatility in Croatia", Economic Research, Vol.20 No.1 ,1-7.

Giannakis B.G.at all (1990) ,"Cumulant-Based order Determination of Non Gaussian ARMA Models, "IEEE Transac. Acoustics , Speech and Signal Processing, Vol. 38,No 8, 1411-1423

Giannakis B.G. and Delopoulos (1995) ," Comulant –based Autocorrelation Estimation of non-Gaussian Linear Process", Signal Processing , Vol.47 ,1-17 .

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance, 48(5), 1779–1801.

Hald A.(2000) ,"The Early History of the Cumulants and Gram-Chalier Series" ,International Statistical Review",Vol.66, No 2,137-153.

Kaiser. T. and Mendel .M.J.,(1995) ,"Finite Sample Covariances of Second-,Third and Fourth-Order Cumulants ",USC-SIPI report#301, University of Southern California

Ling, S. and M. McAleer (2002), "Stationarity and the existence of moments of a family of GARCH processes", Journal of Econometrics, Vol .106, Jan., 109-117.

Miljković V .and Radović O., (2006), "Stylized Facts Of Asset Returns: Case of Belex", Facta Universitatis , Series: Economics and Organization Vol. 3, No 2, 189 – 201.

Pipie M., Bayesian Comparison Of Garch Processes With Skewness Mechanism In Conditional Distributions, (2006) ACTA PHYSICA POLONICA B, Vol. 37 , No 11,

Poon H. and Granger C., (2003), "Forecasting Volatility in Financial market : A review", Journal of Economic Literature, Vol. XLI , June, 478-539.

Poon H. (2005) Forecasting Volatility in Financial Markets: A Practical Guide Wiley Finance Series .

ISBN : 9780974211428

Porat B. and Friendlander B.(1988), ”Performance Analysis Of Parameter Estimation Algorithms Based on Higher Order Moments “, Int.J. Adaptive Control and Signal Processing , Vol. 3. ,No. 3,191-229.

Silvey T.(2007) : “An investigation of the relative performance of Garch models versus simple rules in forecasting volatility” , In John Knight and Stephen Satchel (editors), Forecasting Volatility in the Financial Markets , Third Edition, 101-130, Elsevier Finance, Quantitative Finance Series, Butterworth-Heinemann, Oxford.

Swami A., Mendel J. (1989), ”Closed Form Estimation of MA Coefficients Using Autocorrelations and Third –Order Cumulants”, Vol. 37,No.11,1794-1797.

Swami A., ”System Identification using Cumulants” , (1989), USCI-SIPI report #140, University of Southern California .

Zakoian, J.M. (1994). Threshold heteroskedastic models. Journal of Economic Dynamics Control, Vol.18, No 5, 931–955.