A new approach to Exchange Rate Volatility Forecasting

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Abstract: The aim of this paper is to elucidate a need for the optimization of the two most used methods of exchange rate volatility forecasting: GARCH method based on daily returns and ARMA realized volatility forecasts based on intraday, 30 min, returns. Volatility forecasts of the following closing exchange rates: EUR/USD, USD/JPY and USD/CHF are tested. The sample period is from August 2, 2011 to December 1, 2011.Data is taken from Bloomberg. In the first step of the analysis, GARCH models of different orders are estimated, assuming the GAD distributed series of exchange rate returns. In the second step, realized daily volatilities, defined as the sum of intraday squared returns, are used to estimate ARMA volatility model parameters and to calculate forecasting errors. It was demonstrated ,as anticipated, that high-frequency data enhance the performance of volatility forecasts significantly. Namely, it was found that the intraday realized volatility forecasts outperform GARCH volatility forecasts for all exchange rate series. In the next step a higher order cumulants are used to improve ARMA parameter estimation, as important for both models. A novelty of this research is the finding that the digital whitening of squared returns (GARCH) as well as of realized volatility can be significantly improved by using Higher Order Cumulants -HOC based ARMA parameter estimation. It was suggested to use HOC- Realised volatility as a proxy for real exchange arte volatility.

1. Introduction

In the last decade, the progress in exchange rate volatility modeling has slowed. First , the focus of volatility modeling continues to be mostly univariate. Many multivariate GARCH and stochastic volatility models for time-varying return volatilities and conditional distributions have been proposed (see, for example, the surveys by Bollerslev, Engle and Nelson (1994) and Ghysels, Harvey and Renault (1996)). Nonetheless it is rare to see substantive applications of those multivariate models dealing with more than a few assets simultaneously.
Second , the availability of truly high-frequency intraday data has made limited impact on the modeling of daily return volatility. Indeed, standard practice is still to produce forecasts of daily volatility from daily return observations, even when higher-frequency data are available.
However , exchange rate volatility modeling is important in the framework of international trade due to two main reasons. Firstly, the national governments have realised the impact of this volatility on their own monetary policies and this has been more so for countries where economic growth is driven by export growth. Thus exchange rate fluctuations are regularly monitored by central banks for macroeconomic analysis and market surveillance purposes. Secondly, due to the increasing number of international portfolios, investors and corporate managers have realized that there is an ever increasing need to address risk in the context of exchange rate volatility.
Finance practitioners have largely avoided volatility modeling and forecasting in the higher-dimensional situations of practical relevance, relying instead on GARCH volatility modeling.

Indeed, there are many variants of ARCH/GARCH models which are developed to improve the out-of-sample GARCH volatility forecasting performance. These models have many strong proponents, who believe that these models are currently the best obtainable forecast estimators. However, most empirical studies on the subject in recent years have found no clear-cut results in improving forecasting performances of this class of GARCH models (Poon & Granger (2003) and Carrol & Kearvey (2009)). There are studies which confirm a very low coefficient of determination produced by GARCH models. For instance, Anderson and Bollerslev (1988) show theoretically

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that R2 for a GARCH (1,1) model tends to 1/K, where K stands for the kurtosis of the distribution of stock returns. This means that the highest R2 for Gaussian returns achievable by GARCH models is bounded from above by 1/3. For the exchange rate returns, which have a non-Gaussian distribution, the kurtosis is usually much higher than three, which means that a volatility forecast performance is expected to be worse in practice.

An investigation of the relative performance of GARCH models versus simple rules in forecasting volatility is done by Silvey (2009). While numerous studies have compared the forecasting abilities of historical variance and GARCH models, no clear winner has emerged. In a thorough review of 93 such studies, Poon and Granger (2003) report that only 22 of the studies conclude that historical volatility forecasts out-of-sample future volatility more accurately, while 17 studies find that GARCH models are more accurate. Brooks (1998) used the Dow Jones Industrial Average (DJIA) composite daily data to test in- and out-of-sample forecasts obtained by using GARCH, EGARCH, GRJ and HS (historical volatility) models. The R^2 achieved was around 25% for each of the models. Ederington and Guan (2004) used GARCH, EGARCH, HIS and AGARCH to evaluate forecasts of the DJIA and S&P daily volatilities. They found that absolute returns perform better than squared returns, but without achieving any statistical difference between the performances of the forecasting models.

The theoretical and empirical properties of realized volatility are derived in Andersen, Bollerslev, Diebold and Labys (2001) for foreign exchange. Further empirical evidence is provided in Andersen, Bollerslev, Diebold and Ebens (2001) for U.S. equities. Related research into the econometric properties of realized volatility includes Barndorff-Nielsen and Shephard (2001, 2002a, 2002b) and Areal and Taylor (2002).
There are few comparisons of the forecast accuracy of option prices and the high-frequency challenger called realized volatility. Taylor and Xu (1997) find volatility information in five-minute Mark/$ returns incremental to option implied volatilities, when forecasting volatility one-hour ahead. On the other hand, Blair, Poon and Taylor (2001) claim almost all the useful predictive information is in option prices when forecasting S & P 100 volatility one or more days into the future.

This paper provides the empirical evidence to support the notion that the GARCH-based daily volatility models do not capture stylized facts associated with squared exchange rate returns. We show that the nature of this GARCH controversy does not seem to focus upon the model structure but rather upon the distribution of the squared returns for which second order moments and higher order moments of order zero do not represent a sufficient statistic for the parameter estimation. The evidence is based on the monthly time series data of real daily exchange rates for the period from September 1, 2006 to September 1, 2009. We show the enhanced performance of the realised volatility forecasts, relative to GARCH volatilities, which comes from the use of high frequency returns.

The organization of the paper is as follows: the first section briefly summarizes the literature findings. The second section introduces the problem and shows the data description of the currencies used; the third section presents the GED/GARCH model results and respective coefficients of determination and calculates the GARCH volatilities. Furthermore, it presents realized volatility forecasts and compare them with GARCH forecasts. The last section identifies where the problem of GARCH volatility modeling is, and introduce ARMA parameter estimation based on Higher Order Cumulants (not only skewness and kurtosis) to capture the non-used information available in the data.

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2. The Problem and the Model

The statistical properties of exchange rate returns, common across a wide range of developed stock markets and time periods, are called stylized facts. The stylized statistical properties of asset returns of developed markets are analyzed empirically and subsequently summarized by Cont (2001). They include the following findings:

The autocorrelations of asset returns are often insignificant, except for high frequency data ($f \simeq 20$ minutes or less); Heavy tails, with a finite tail index, which is higher than two and lower than five for most data sets studied; Gain/loss asymmetry: one observes large drawdown in stock prices and stock index values but not equally large upward movements; Aggregational Gaussianity according to the Central Limit Theorem; Volatility clustering, namely, different measures of volatility display a positive autocorrelation over several days, which quantifies the fact that high-volatility events tend to cluster in time; Conditional heavy tails even after correcting the returns for the volatility clustering ;Slow decay of autocorrelation in absolute or squared returns, i.e. non-stationarity; average effect; most measures of the volatility of an asset are negatively correlated with the returns of that asset; Volume/volatility correlation i.e., the trading volume is correlated with all measures of volatility.

The fact that exchange rate returns are often characterized by volatility clustering—which means that periods of a high volatility are followed by periods of a high volatility and periods of a low volatility are followed by periods of a low volatility—implies that the past volatility could be used as a predictor of the volatility in the next periods. As an indication of volatility clustering, squared returns often have significant autocorrelations and consequently can be modeled by using the well-known GARCH model.

Let e_t denote a discrete time stationary stochastic process. The GARCH (p,q) (Generalized Autoregressive Moving Average Conditional Heteroskedasticity) process is given by the following set of equations (Bollerslev, 1986, 42-56):

$$r_t = \log(p_t) - \log(p_{t-1}) \tag{1}$$

$$r_t = x_{(k)} g_{(k)} + e_t \tag{2}$$

$$e_t = v_t \sqrt{h_t}$$

$$e_t / _{t-1} \approx N(0, h_t), \tag{3}$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i e^2_{t-i} + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{4}$$

in which p_t represents stock prices, e_t represents random returns, $x_{(k)}$ is a vector of explanatory variables, $g_{(k)}$ is a vector of multiple regression parameters, h_t is the conditional volatility, α_i is autoregressive, and β_j is the moving average parameter as related to the squared stock market index residuals.

An equivalent ARMA representation of the GARCH (p, q) model is given by:

$$e_t^2 = \alpha_0 + \sum_{i=1}^{P} (\alpha i + \beta_i) e^2_{t-i} + \nu_t - \sum_{j=1}^{q} \beta_j \nu_{t-j} \tag{5}$$

where $\nu t = e_t^2 - h_t$ and, by definition, it has the characteristics of (i.i.d) white noise.

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In other words, the GARCH (p, q) volatility model is an Autoregressive Moving Average (ARMA) model in e_t^2 driven by white noise ν_t which is not necessarily Gaussian.

From Time Series Analysis it emerges that GRACH volatility, by definition , is nothing else but a prediction of squared FX returns . Thus it is unlikely that GARCH model can produce a reliable proxy for exchange rate volatility. Nonetheless, criticisms of GARCH models had begun to gain significant acceptance early 2000 as they were able to make a credible case that GARCH latent volatility no longer reflected a real volatility.

Although GARCH's reputation has fluctuated since 2000 , few researchers would deny that his GARCH model is one of the greatest and most influential models ever made on the subject of volatility . We argue for a much more important role for the higher order statistics , which should adjust ARMA parameter estimates in order to ensure (relative) white residuals which otherwise would be a series of non Gaussian noise

In the case of exchange rate returns, there is no trend; driving white noise is not, most commonly, Gaussian and subsequently the second order moment of the associated probability density distribution is not a sufficient statistic for the ARMA parameter estimation. In fact, it is well known that for a non-Gaussian process, a higher order moment exists and is different from zero. The hypothesis in this article is that higher order moments of GARCH forecasting errors contain the information necessary to capture heavy tails and volatility clustering.

3. Empirical Analysis

GARCH and Realised Volatility forecasts

The intention of this study is to use the GED/GARCH model for measuring volatility and apply it to exchange rate returns. The empirical analysis is based on daily quotations of opening daily exchange rates for the following currency pairs: CHF/EUR, CHF/JPY, EUR/JPY, GBP/EUR, USD/CHF, USD/EUR and EUR/TRY during the period from September 1, 2006 to September 1, 2009, taken from the OANDA and ECB online databases. The common sample of exchange rate descriptors is presented in Table 1.

The evaluation and comparison of volatility models is made difficult by the fact that the conditional variance is unobservable. This has made it difficult to identify poor models, and may explain that so many volatility models have been able to coexist. The first approach to circumvent this problem, was to substitute squared returns for the unobserved conditional variance

Table 1: Exchange rates for the period 09/01/2006 to 09/01/2009. *Sources: ECB and OAND data bases.*

	CHF/EUR	CHF/JPY	EUR/TRY	EUR/JPY	GBP/EUR	USD/EUR	USD/CHF
Mean	0.630	0.943	1.991	1.501	1.321	0.721	1.145
Median	0.622	0.971	2.048	1.565	1.306	0.729	1.153
Maximum	0.691	1.047	2.312	1.695	1.527	0.801	1.274
Minimum	0.596	0.762	1.668	1.144	1.027	0.627	0.984
Std. Dev.	0.023	0.070	0.174	0.156	0.144	0.048	0.074
Skewness	0.566	-0.895	-0.190	-0.768	-0.196	-0.329	-0.218
Kurtosis	2.063	2.757	1.535	2.180	1.623	2.087	1.896
Jarque-Bera	97.152	146.972	103.069	136.454	92.300	57.078	63.392
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum	680.675	1018.365	2150.537	1621.411	1426.343	778.742	1236.561
Sum Sq. Dev.	0.566	5.312	32.668	26.371	22.516	2.435	5.851
Observations	1080	1080	1080	1080	1080	1080	1080

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The common sample description of squared returns (r^2_t) is presented in Table 2. it is based on Exchange rates for the period 09/01/2006 to 09/01/2009. *Sources: ECB and OANDA*.

Table 2: Sample descriptors for squared returns.

	R^2CHF/EUR	R^2CHF/JPY	R^2EUR/TRY	R^2EUR/JPY	R^2GBP/EUR	R^2USD/CHF	R^2USD/EUR
Mean	0.084686	0.319554	0.904712	0.47254	0.109851	0.256933	0.24204
Median	0.01249	0.04419	0.218768	0.068949	0.008741	0.04274	0.036277
Maximum	7.330931	15.12435	43.2685	27.00682	8.040269	14.6517	11.96905
Minimum	0	0	0	0	-3.223591	0	0
Std. Dev.	0.327323	0.966889	2.598238	1.588638	0.547596	0.848209	0.72274
Skewness	13.12559	8.50038	9.355778	10.44167	7.483825	11.47573	8.362307
Kurtosis	246.0965	101.6348	121.9818	147.1139	92.93137	174.886	101.1779
Jarque-Bera	2687836	450385.9	652201.4	953338.6	373679.4	1351968	445924.2
Probability	0	0	0	0	0	0	0
Sum	91.37667	344.7991	976.1838	509.8704	118.5296	277.2307	261.1613
Sum Sq. Dev.	115.4971	1007.795	7277.407	2720.626	323.25	775.5764	563.096
Observations	1079	1079	1079	1079	1079	1079	1079

The table shows that all the variables are non-Gaussian (according to the skewness, kurtosis, and the Jarque-Bera test for normality).

The best significant models found for the squared returns using E-Views for GED distribution, together with the respective coefficients of determination, are presented in Table 3. It can be seen from the table that the maximum coefficient of determination achieved was 23%. This confirms a low explanatory power of the GARCH model in the case of exchange rate volatility.

Table 3: GARCH-ARMA

GARCH/ARMA models		α1	α2	α3	α4	α5	β1	β2	β3	β4	β5	R^2
GBP/EUR	Coeff	0.386	0.623	0.709	-0.722		-0.492	-0.712	-0.628	0.864		**11.54%**
	St. error	0.048	0.020	0.020	0.048		0.034	0.017	0.016	0.032		
CHF/JPY	Coeff	0.844	0.227	-0.753	0.666		-0.530	-0.357	0.578	-0.544		**23.00%**
	St. error	0.090	0.073	0.090	0.060		0.094	0.073	0.091	0.064		
USD/CHF	Coeff.	0.726	-0.064	-0.029	0.327	0.038	-0.256	-0.127	-0.039	-0.334	-0.200	**21.77%**
	St. error	0.404	0.587	0.589	0.473	0.155	0.403	0.450	0.453	0.291	0.081	
USD/JPY	Coeff.	1.075	-0.732	0.649			-0.914	0.673	-0.672			**9.55%**
	St. error	0.130	0.185	0.103			0.123	0.156	0.086			
EUR/JPY	Coeff.	0.978					-0.861					**17.46%**
	St. error	0.008					0.021					
CHF/EUR	Coeff.	0.112	-0.089	0.963			-0.061	0.143	-0.962			**7.45%**
	St. error	0.009	0.009	0.009			0.011	0.010	0.011			
USD/EUR	Coeff.	2.014	-1.674	0.659			-1.781	1.348	-0.541			**16.12%**
	St. error	0.117	0.174	0.077			0.121	0.161	0.078			
EUR/TRY	Coeff.	-0.199	0.127	-0.236	0.254	0.829	0.375		0.415	-0.081	-0.742	**11.48%**
	St. error	0.089	0.035	0.074	0.037	0.042	0.044		0.092	0.047	0.076	

GARCH variances, h_t, t=1, 2 ... n, based on daily returns .are calculated by using ARMA representations of the GARCH models. Residuals from those models are calculated using the parameters presented in the table 3. Finally, GARCH variance is obtained by using the formula

$$h_t = r^2 - \nu_t, \ t=1, 2... , \qquad (6)$$

Realized volatility, defined as the sum of intraday ,30 min squared returns, provides a more accurate estimate of the latent process that defines volatility than is given by daily squared returns .

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$$RV==\sum_{1}^{48} r^{2}_{t-i} \qquad (7)$$

The theoretical and empirical properties of realized volatility are derived in Andersen, Bollerslev, Diebold and Labys (2001) for foreign exchange. Further empirical evidence is provided in Andersen, Bollerslev, Diebold and Ebens (2001) for U.S. equities.

The parameters of the both models related to EUR/USD exchange rates are presented in the tables 4 and 5.

Table 4 :Realised HF daily volatility

Dependent Variable: RV				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.337	0.148	9.012	0
AR(2)	1.320	0.095	13.927	0
AR(3)	0.064	0.049	1.700	0.02
AR(4)	-0.590	0.086	-6.856	0
MA(2)	-1.013	0.066	-15.456	0
MA(6)	0.498	0.036	13.924	0
R-squared	0.277573	Mean dependent va		1.344753

Table 5 :ARMA GARCH daily volatility

Dependent Variable: r2				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.067	0.024	2.777	0.0069
AR(1)	-0.252	0.066	-3.834	0.0003
AR(2)	-0.393	0.051	-7.704	0
AR(3)	-0.837	0.063	-13.188	0
MA(1)	0.403	0.033	12.384	0
MA(2)	0.414	0.029	14.275	0
MA(3)	0.972	0.029	33.623	0
R-squared	0.22379	Mean dependent va		0.074051

Based on the results in tables 4 and 5 , forecast are estimated and presented in Figure 2. Results imply that GARCH volatility forecasts underestimates volatility .

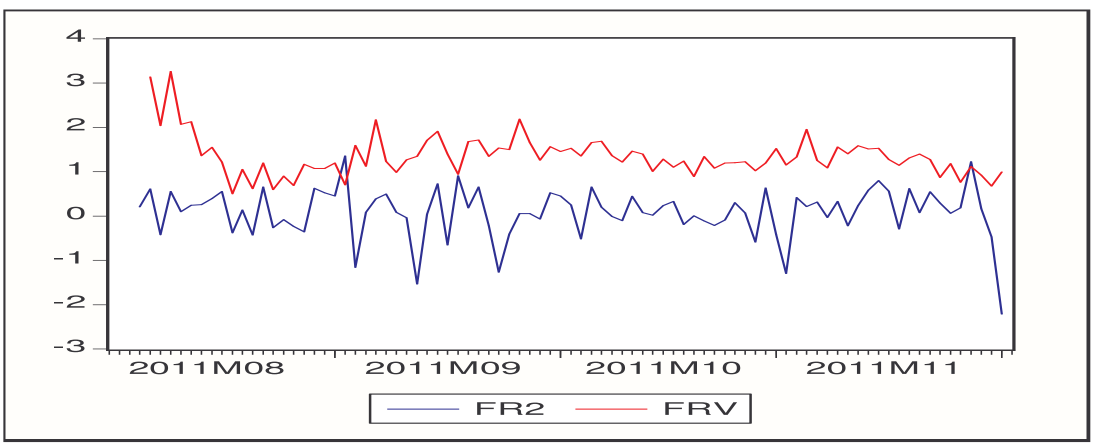

Figure 2:Volatility forecasts

The performance of both volatility forecasting methods is assessed by testing information content in the residuals resulting from these methods. Statistical description of residuals is presented in the table 6.

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Table 6 : Statistical description of the forecasting errors

	RESR2	RESRV	R2	RV
Mean	0.004979	-0.00747	0.071895	1.356771
Median	-0.0283	-0.06564	0.006483	1.186062
Maximum	1.175881	2.214157	1.432809	3.206
Minimum	-0.25228	-0.8025	0	0.465526
Std. Dev.	0.190023	0.543144	0.212333	0.63047
Skewness	3.428123	1.559967	4.386001	1.085673
Kurtosis	19.69719	6.394839	24.15063	3.804129
Jarque-Be	1153.89	75.29203	1922.423	19.65833
Probability	0	0	0	0.000054
Sum	0.423201	-0.63505	6.326732	119.3958
Sum Sq. D	3.033146	24.78046	3.922416	34.58185

Results in table 6 further imply that none of methods based on the second order moments produces white noise residuals . Thus, high kurtosis and skewness, demonstrate existence of the information contained in the higher order cumulants .

Therefore, the GARCH volatility forecasting errors are further used to calculate the higher order moments. The cumulants of order four , C^4 ($\tau1,\tau2,\tau3$), are then calculated using the following equation:

$$C^4(\tau1,\tau2,\tau3)= (\sum(x(t)x(t+\tau_1)x(t+\tau_2)\ x(t+\tau_3))/n -$$

$$+C^2_x(\tau1)\ C_x(\tau2-\tau3) - C^2_x(\tau2)\ C_x(\tau3-\tau1)-C^2_x(\tau3)\ C_x(\tau1-\tau2), \quad (9)$$

where n is a number of observations and where the second-order cumulant C^2_x (τ) is just the autocorrelation function of the time series x_t. The zero lag cumulant of order 3 C^3_x (0, 0) normalized by σ_x^3 has the skewness γ^3_x; C^4_x (0, 0, 0) normalized by σ_x^4 is known as kurtosis γ^4_x.
A new method of the AR parameter estimation for non-Gaussian ARMA (p,q) processes is based on the modified Yule-Walker system where autocorrelations are replaced by third or fourth order cumulants:

$$\sum_{1=1}^{p} \alpha i\ C^3(k-i,k-l) \quad = - \quad C^3(k,k-l) \qquad k\geq l\geq q+1 \qquad (10)$$

$$\sum_{1=1}^{p} \alpha i\ C^4(k-i,k-l,\ k-m) \quad = - \quad C^4(k,k-l,\ k-m) \qquad k\geq l\geq m\geq q+1 \qquad (11)$$

The efficient MA parameter estimation can be performed by applying one of the proposed algorithms, for instance, q-slice algorithm (Swami 1989). Q –slice algorithm uses autoregressive residuals calculated after estimating the AR parameters or ARMA (10). Following up, the impulse response parameters ψ_I of the pure MA model of x_t model are then estimated using cumulants (12):

$$x_t=\sum_{0}^{\infty}\psi_j a_{t-j} \qquad i=1.2\ldots\infty \qquad (11a)$$

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$$\psi j = \frac{\sum_{1}^{p} \alpha i \, C^3(q-i,j)}{\sum_{1}^{p} \alpha i \, C^3(q-i,0)} \qquad j=1,2\ldots q \qquad (12)$$

Or by using :

$$\psi j = \frac{\sum_{1}^{p} \alpha i \, C^4(q-i,j,0)}{\sum_{1}^{p} \alpha i \, C^4(q-i,0,0)} \qquad j=1,2\ldots q \qquad (13)$$

The MA parameters of the ARMA model are obtained by means of the well known relationship:

$$\beta_j = \sum_{0}^{p} \alpha i \, \psi_{(j-i)} \qquad j=1,2\ldots q \qquad (14)$$

The cumulants based ARMA estimates are shown to be asymptotically optimal by Friendler B. and Porat B. (1989).

The cumulants of the residuals obtained from squared returns and from realised volatility are presented in Figure 3

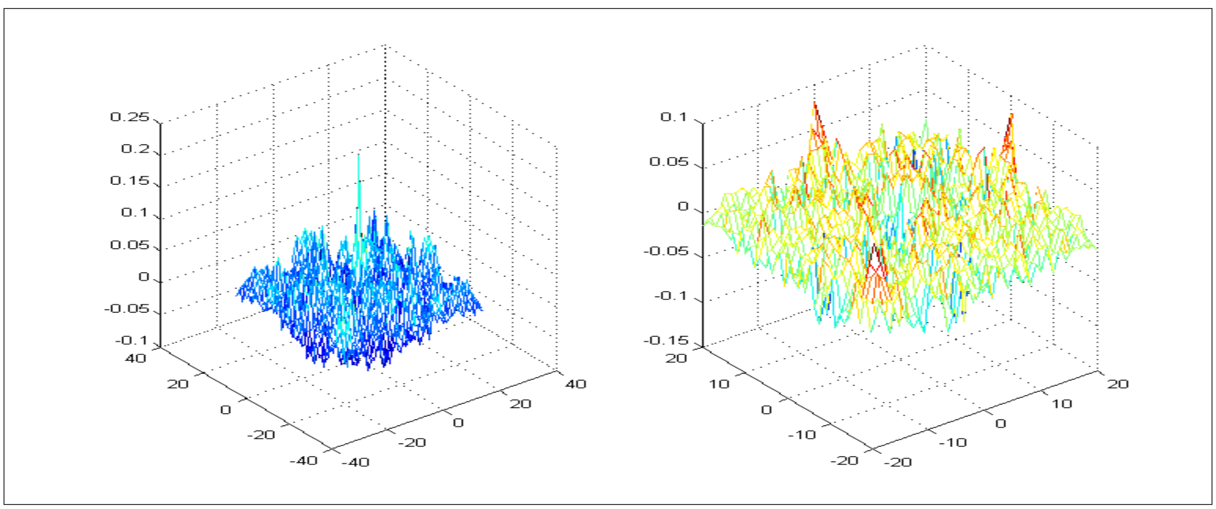

Figure 3: Fourth-order cumulants of the GARCH and Realised volatility forecasting errors –EUR/USD.

From Figure 3, it can be seen that the higher order cumulants (HOC) persist to be different from zero for different lags, which demonstrates that there is significant information in the both series ,which has not been extracted by the model. Engle &Patton (2000) also point out that heteroskedasticity of returns, r_t , implies (even more) heteroskedasticity in the squared returns, $r2_t$. So parameters are estimated inefficiently and the usual standard errors are misleading.

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Instead ,after applying HOC-ARMA estimation method r^2 and RV residuals became flat as it can be seen from Figure 4.

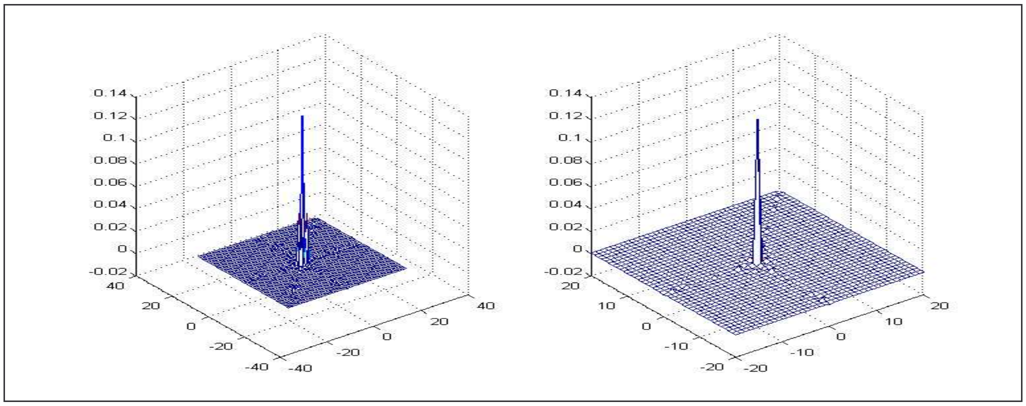

Figure 4: Fourth-order cumulants of the HOC GARCH and HOC Realised volatility forecasting errors –EUR/USD.

Therefore we strongly recommend using the HOC-based ARMA parameter estimation for improving forecasting efficiency..

4. Conclusion

From Time Series Analysis it emerges that GRACH volatility, by definition , is nothing else but a prediction of the squared FX returns . Thus, it seems unlikely that GARCH model can produce a reliable proxy for a daily exchange rate volatility. However ,criticisms of GARCH models had begun to gain significant acceptance only early 2000, as researchers were able to make a credible case that GARCH latent volatility no longer reflected a real volatility. Although GARCH's reputation has fluctuated since 2000, few researchers would deny that the GARCH model is one of the most influential models ever made on the subject of volatility .

This paper argues for a much more important role for the higher order statistics in volatility forecasting , which should adjust ARMA parameter estimates in order to ensure relatively white residuals which otherwise would be a series of non Gaussian noise with cumulants different from zero.

It has been demonstrated throughout this empirical analysis that GARCH-type models do not explain more than 30% of the exchange rate volatility, a purpose which is better served through the use of the HOC-based ARMA/GARCH parameter estimation method.

However ,our results also indicate that using high-frequency data enhances the performance of realized volatility forecasts significantly. We find that the intraday realized volatility forecasts outperform GARCH volatility forecasts for all exchange rate series. Notwithstanding importance of HF data , the existence of t higher order cumulants in forecasting errors , revealed that neither ARMA -GARCH model not realised volatility model, can capture sufficient information from given time series .

It is further demonstrated that both volatility forecasting method can be significantly improved by using HOC based ARMA parameter estimation. . Thus, we propose that future academic research studies in this area focus on the question of whether the covariance function, skewness, and kurtosis constitute sufficient statistics for exchange rate volatility estimation.

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