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Capturing Stylized Facts of Stock Market Volatility and Higher Order Cumulant Function

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Abstract .The aim of this paper is to explore how the stylized facts of a stock market (SM) return can be captured by using the Higher Order Cumulant (HOC) function. Two models are tested: GARCH and Realized Volatility (RV). ARMA- GARCH and ARMA-RV produce forecasting errors which are not Gaussian. The ability of these models to whiten higher order cumulant function is tested empirically, by using: daily closing stock market indexes, from June 5, 2010 to February 5, 2013; high frequency exchange rates: EUR/USD, USD/JPY, GBP/USD and USD/CHF during the period from June 3, 2012 to December 1, 2012, taken from Bloomberg; daily closing spot prices taken from the OANDA data base and daily closing indexes also from Bloomberg. In the first step of the analysis, GARCH models of different orders are empirically fitted, assuming the GAD distributed series of squared stock market and exchange rate returns, to see if they can capture all necessary information needed to forecast volatility. Similarly realized daily volatilities, defined as the average of intraday 30 min returns, are used to estimate RV -ARMA volatility model and to calculate innovations. Given the fact that residuals from both models are persistently non Gaussian, in the third step, higher order residual cumulants are calculated and compared with the empirical cumulants of the squared returns. It is further demonstrated that the ability to capture stylized facts of squared stock market and FX returns is significantly improved if ARMA parameter estimation is based on HOC function. It is concluded that the second, third and the fourth order cumulnat functions constitute a sufficient statistics for stock market volatility estimation.

Key Words : Stock Market Volatility Modeling, GARCH model, RV model, ARMA Parameter estimation, Higher Order Cumulants, Non Gaussian distribution, Stylized facts, Sufficient statistics.

JEL Classification: G22

1. Introduction and Literature Review

Amid current financial crisis, corporate managers and investors have realized that there is an ever increasing need to address risk in the context of stock market volatility. There is a long standing discussion about the origins of the financial crisis in general and the currency crisis in particular. In terms of financial economics, an important empirical condition which bears witnesses of the crisis is leptokurticity of real stock market data distribution. Volatility is central to financial theory and investment decisions. Volatility forecasts of stock price are crucial inputs for pricing derivatives as well as trading and hedging strategies. Basic mean variance analysis also requires estimates of the variance for the assets under consideration. Given these facts, the quest for precise forecasts appears to still be ongoing.

Today there is no doubt that the theoretical model of classical economics, based on hypothetical market efficiency, cannot be confirmed by actual market data (Mandelbrot, 1963). His finding was confirmed by generations of researchers. However, this result failed to immanently inspire a concerted effort to develop a better theory. Given the importance of the variance in finance, for example, very little research has examined the precision of variance estimates outside of the unrealistic iid normal assumption for asset returns.

The stylized facts (C. G. de Vries and K. U. Leuvenin 1996) can be classified into three groups. First, several facts constitute so called no (possibility of) arbitrage conditions. They consequently

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have direct economic content. Second, other facts are mere statistical regularities for which we currently lack a good economic explanation. A third category comprises some negative results, artifacts say, i.e. regularities which are commonly hypothesized but for which not much empirical support has been found.

This paper focus on the second group of stylized facts, which have a sound statistical basis, but for which no convincing economic explanation has been established. They are:

- Volatility Clustering and long memory in absolute values of returns; A Time series f absolute values of returns is characterized by important autocorrelation, and the autocorrelation function (ACF) decays slowly with time lags (slower than geometric decay). Long periods of high and low volatility are observed (Bollerslev et al., 1992; Ding et al., 1993; Ding and Granger, 1996).

- Fat Tail Phenomena (Cont 2001); Exchange rate returns, irrespective of the regime, when standardized by their scale, exhibit more probability mass in the tails than distributions such as the standard normal distribution. Freely speaking, this means that extremely high and low realizations occur more frequently than under the hypothesis of normality. A related fact is that the density of the returns is more peaked than the normal density. A popular measure for this latter fact is the kurtosis.

- Skewness; Stock market and exchange rate returns of currencies which experience similar monetary policies exhibit no significant skewness, while dissimilar policies tend to generate skewness. This is often caused by a disparity between monetary policies, such as a hyperinflation versus a deflationary policy.

- Leverage effect in the dynamic structure of volatility. Positive and negative returns of the same magnitude, observed over the past period, have different effects on the current volatility (asymmetry). Current returns and future volatility are negatively correlated (leverage). Presence of the leverage effect implies the asymmetry but the inverse does not hold (Black, 1976).

- Taylor effect; It states : if yt is the series of returns and acf(θ, k) represents the sample autocorrelation of the order k of |yt|θ then the Taylor effect is defined as acf(1,k) > acf(θ,k) for any θ different from 1, Granger and Ding (1993).

The best known models developed for volatility forecasting are the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and the Autoregressive Stochastic Volatility model ARSV model.

A popular ways of comparing volatility models have been: to compare Mean Forecast Errors produced by those models; to estimate a number of models by maximum likelihood and observe which one has the highest log-likelihood value; to use AIC or BIC criteria. Most recently the other approach is taken. Namely, given a set of characteristic features or stylized facts described above, one may ask the following question: "Have popular volatility models been parameterized in such a way that they can accommodate and explain the most common stylized facts visible in the data?" Models for which the answer is positive may be viewed as suitable for practical use. For example, Teräsvirta (1996) investigated the ability of the GARCH model to reproduce series with high

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kurtosis and, at the same time, positive but low and slowly decreasing autocorrelations of squared observations. Carnero, Peña & Ruiz (2004) compared the ARSV model and the GARCH model using the kurtosis –autocorrelation relationship in squared returns as their benchmark. Bai, Russell & Tiao (2003) also compared GARCH and ARSV models. Malmsten &Teräsvirta (2004) discussed this stylized fact in connection with the GRACH and ARSV model. Their paper contains an application of a novel method of obtaining confidence regions for the kurtosis-autocorrelation relationship. The exact representation of kurtosis is derived for both GARCH and stochastic volatility models:

$\rho(n) = (\alpha +_1 \beta_1)^{n-1} * \alpha_1 \nu^2 (1 - \beta 1^2 \beta_1 \alpha_1 \nu^2) / (1 - \beta_1 2 - 2\beta_1 \alpha_1 \nu^2)$, n>=1

$\kappa 4(e_t) = 3(1 - (\alpha_1 + \beta_1 1)^2 / (1 - (\alpha_1{}^2 2\nu\beta_1 \nu^2 + \beta_1{}^2)$, where $E(z_t{}^4) = \nu^4$ and $\kappa 4 = \nu^4 / \nu_2{}^2$ is kurtosis of returns

It was demonstrated both analytically and empirically that the GARCH (1, 1) model with starting autocorrelation of squares was observed in a large number of financial series. It was also found that "the first-order autocorrelations", given the certain kurtosis, are lower in the ARSV than the EGARCH model with normal errors". This may, at least to a certain extent, explain the fact the ARSV (1) model seems to fit the data better than its EGARCH or GARCH counterpart. However, the skewness of the squared returns which is frequently encountered in stock market variables, cannot be reproduced by any of the existing volatility models.

A conclusion that emerges from those considerations, which are largely based on results on the moment structure of these models, is that "none of the models dominates the others when it comes to reproducing stylized facts in typical financial time series". By comparing the difference between the theoretical GARCH kurtosis and the estimated kurtosis, it is demonstrated how t distribution assumption adds to the flexibility of the GARCH model and helps the model parameters to reproduce, in a better way, the stylized fact of high kurtosis and relative low autocorrelations of squared observations. Partial empirical improvements are made only when considering t or GED distribution of squared returns. Nonetheless, it is to be noted that the estimator being used plays a significant role when comparing theoretical and empirical kurtosis. What is "sufficient statistics" for ARMA parameter estimation, in the case of SM data, remains again a key problem which has not been solved yet.

Both volatility clustering and conditional non-normality can induce the leptokurtosis which is typically observed in financial data. Bai &Russell (2001) found theoretically and empirically that, for GARCH and ARSV models, volatility clustering and non-normality contribute interactively and symmetrically to the overall kurtosis of the series of squared returns (e_t):

$e_t = z_t \sqrt{h_t}$

$\kappa_4(e_t) = (\kappa_4(g_t) + \kappa_4(z_t) + (5/6) * \kappa_4(g_t)\kappa_4(z_t)) / (1 - (1/6) * \kappa_4(g_t)\kappa_4(z_t)),$

where $\kappa_4(g_t)$ is GARCH kurtosis, $\kappa_4(z_t)$ is a standard Normal distribution kurtosis.

The aim of this paper is to take a new direction which leads back to the essence of Time Series Analysis. Namely, it is argued that the sufficient statistics for SM returns is defined in terms of the higher order cumulant (HOC) function. It is hypothesized that the volatility model extracts the information about the stylized facts, contained in the squared returns, better if ARMA parameters are calculated by using both second, third and fourth order cumulant functions.

That is to say that those stylized facts are not seen only in autocorrelation function, kurtosis and skewness of squared returns, but also in the third and the fourth order cumulant functions mentioned above.

The empirical analysis is aimed to show that the model parameters based on HOC more successfully flatten the second, the third and the fourth order cumulant functions of the model returns. The analysis refers to ARMA GARCH and ARMA –RV Volatility models.
The paper has the following contents: The higher order cumulants and the notion of "sufficient statists" are defined in Section 2; The ARMA parameter estimation based on the modified Yule Walker difference equations is introduced in Section 3; Section 4 presents empirical findings and Section 5 contains conclusions.

2. The Problem and the Model

The fact that stock market returns are often characterized by volatility clustering—which means that periods of a high volatility are followed by periods of a high volatility and periods of a low volatility are followed by periods of a low volatility—implies that the past volatility could be used as a predictor of the volatility in the next periods. As an indication of volatility clustering, squared returns often have significant autocorrelations and consequently can be modeled by using the well-known GARCH model.

Let e_t denote a discrete time stationary stochastic process. The GARCH (p,q) (Generalized Autoregressive Moving Average Conditional Heteroskedasticity) process is given by the following set of equations (Bollerslev, 1986, 42-56):

$$r_t = \log(p_t) - \log(p_{t-1}) \qquad (1)$$

$$r_t = x_{(k)} g_{(k)} + e_t \qquad (2)$$

$$e_t = z_t \sqrt{h_t}$$

$$e_t /_{t-1} \approx N(0, h_t), \qquad (3)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i e^2_{t-i} + \sum_{j=1}^{q} \beta_j h_{t-j} \qquad (4)$$

in which p_t represents stock prices, e_t represents random returns, $x_{(k)}$ is a vector of explanatory variables, $g_{(k)}$ is a vector of multiple regression parameters, h_t is the conditional volatility, α_i is autoregressive, and β_j is the moving average parameter as related to the squared stock market index residuals.

An equivalent ARMA representation of the GARCH (p, q) model is given by:

P q

 ISBN : 9780974211428

$$e_t^2 = \alpha_0 + \sum_{i=1} (\alpha i+\beta_i)e^2_{t-i} + \nu_t - \sum_{j=1} \beta_j \nu_{t-j} \quad (5)$$

where $\nu t = e_t^2 - h_t$ and, by definition, it has the characteristics of (i.i.d) white noise.

In other words, the GARCH (p, q) volatility model is an Autoregressive Moving Average (ARMA) model in e_t^2 driven by white noise ν_t .The e_t^2 Is stationary if $(\alpha i+\beta_i) < 1$

3. Parameter estimation using HOC

Bai, Russell, and Tiao (2003) attribute the leptokurtosis in the financial data to both volatility clustering and conditional non-normality. They showed that the GARCH specification can generate the excess kurtosis observed in most of the financial data. Hall and Yao (2003) state that for heavy-tailed errors, the asymptotic distributions of quasi-maximum likelihood parameter estimators in ARCH and GARCH models are non-normal and are particularly difficult to estimate using standard maximum likelihood method based on the second order moments.

During last two decades dynamic form of higher order cumulants (lag≠0) have been used in many fields: e.g., signal data processing, adaptive filtering, harmonic retrieval biomedicine and image reconstruction. Unbelievable as it may seem, they have not been used in economics and finance. Nevertheless there were trials in finance to use central moments, like co- skeweness and co-kurtosis to build Capital Asset Price model.

In the signal-processing research community, a great deal of progress in higher-order statistics (HOS) began in the mid-1980s. These last 20 years have witnessed a large number of theoretical developments as well as real applications. Blind Estimation Using Higher-Order Statistics focuses on the blind estimation area and records some of the major developments in this field. For example, in the area of digital signal processing, Giannakis (1987) was first to show that AR parameters of non Gaussian ARMA signals can be calculated using third and fourth order empirical cumulants

$$C^3_x(\tau 1,\tau 2)= (\sum(x(t)x(t+\tau_1)x(t+\tau_2))/(n- \tau 1-\tau 2 \ ) \quad (6)$$

$$C^4(\tau 1,\tau 2,\tau 3)= (\sum(x(t)x(t+\tau_1)x(t+\tau_2)\, x(t+\tau_3))/n - \tau 1-\tau 2-\tau 3)$$

$$-C^2_x(\tau 1)\, C_x(\tau 2-\tau 3) - C^2_x(\tau 2)\, C_x(\tau 3-\tau 1)-C^2_x(\tau 3)\, C_x(\tau 1-\tau 2) \quad (7)$$

where n is a number of observations and where the second order cumulant $C^2_x(\tau)$ is just the autocorrelation function of the time series x_t. The zero lag cumulant of the order3 $C^3_x(0,0)$ normalized by σ_x^3 is skewness γ^3_x; $C^4_x(0,0,0)$ normalized by σ_x^4 is known as kurtosis γ^4_x.. The initially developed algorithms for ARMA –HOC parameter estimation were not stable and did not provide consistent estimates.

 ISBN : 9780974211428

Oyet A. (2000, pg 4) proved that efficient ARMA parameters can be obtained by using a modified set of Yule Walker equations where autocorrelations are replaced by third or fourth order cumulants (Gianninakis -1990) :

$$\sum_{1=1}^{p} \alpha i\, C^3(k-i,k-l) = -\ C^3(k,k-l) \qquad k \geq l \geq q+1 \qquad (8)$$

$$\sum_{1=1}^{p} \alpha i\, C^4(k-i,k-l,\ k-m) = -\ C^4(k,k-l,\ k-m) \qquad k \geq l \geq\ m \geq q+1 \qquad (9)$$

Swami [24,25] developed the MATLAB routine AREST which enable AR parameter estimation using both the second and the third order cumulants .

Once the AR residuals are calculated, the MA parameters can be calculated by using the routine MAEST which uses the least squares set of equations:

$$\sum_{1}^{q} \beta_i\, C^3(n-i,n-i) - \sum_{1}^{q} \beta_i^{\,2} C^2(n-i) = C^2(n)\ ,\ N= -q\ldots,2q \qquad (10)$$

where both second and third order cumulants are used.

3. Empirical Analysis

The ability of the ARMA-GARCH parameter estimation method to whiten higher order cumulant function is tested empirically by using daily closing stock market indexes, from June 5, 2010 to February 5, 2013; by using high frequency exchange rates: EUR/USD, USD/JPY, GBP/USD and USD/CHF during the period from June 3, 2012 to December 1, 2012, taken from Bloomberg and daily closing spot prices taken from the OANDA data base and daily closing indexes also from Bloomberg.

The sample statistics of exchange rate squared returns is presented in Table 1 while the sample statistics for the squared index returns is presented in Table 2.

Table 1: Sample description for squared returns on daily exchange rates for the given period.

	R2EURUSD	R2EURJPY	R2USDCHF	R2GBPUSD
Mean	0.0510	0.0614	0.1724	0.0235
Median	0.0148	0.0163	0.0280	0.0106
Maximum	0.6462	0.8801	3.8756	0.4532
Minimum	0.0000	0.0000	0.0000	0.0000
Std. Dev.	0.0904	0.1184	0.5174	0.0462
Skewness	3.6131	4.1007	5.4974	6.8811
Kurtosis	19.9532	24.6778	35.2496	63.2465
Jarque-Bera	1712.2990	2708.3320	5852.9730	19254.3400

 ISBN : 9780974211428

The table shows that all the variables are non-Gaussian (according to the skewness, kurtosis, and the Jarque-Bera test for normality).

Table 2*: Stock market returns-Sample description:*

	RSMI	RDJ	RSP500	RNSQ	RDAX	RFTSE100	RNIK	RBSE	SMIVOL
Mean	0.01	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.07
Median	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maximum	2.10	1.80	2.00	2.10	2.30	2.20	2.40	23.60	57.20
Minimum	-1.80	-2.50	-3.00	-2.70	-2.60	-2.10	-4.80	-29.40	-91.60
Std. Dev.	0.44	0.46	0.51	0.54	0.62	0.49	0.57	1.47	15.69
Skewness	-0.21	-0.40	-0.38	-0.23	-0.13	-0.15	-0.88	-5.32	-0.45
Kurtosis	6.50	6.59	6.83	5.44	5.26	4.92	10.35	313.00	6.62
Jarque-Bera	378.688	412.7403	465.092	188.5072	157.4913	115.1577	1742.022	2934448	423.7915
Observations	732	732	732	732	732	732	732	732	732

The most significant models found for the exchange rate squared returns by using E-Views for GED distribution, together with the respective coefficients of determination, are presented in Table 3.

Table 3: GARCH-ARMA estimates.

GARCH/ARMA models		α1	α2	α3	α4	β1	β2	β3	β4	R^2
R2EUR/USD	Coeff	-0.445	-0.818	-0.709		0.613	0.797	0.843		0.144
	St. error	0.181	0.049	0.178		0.135	0.033	0.146		
R2EUR/JPY	Coeff		0.457			0.228	-0.885	-0.165	0.311	0.349
	St. error		0.086			0.085	0.095	0.085	0.064	
R2USD/CHF	Coeff.	-0.333	0.258			0.927				0.274
	St. error	0.112	0.105			0.057				
R2GBP/USD	Coeff.	-0.087	0.840				-0.981			0.067
	St. error	0.050	0.055				0.023			

It can be seen from the table that the maximum coefficient of determination achieved is 34%. This confirms a low explanatory power of the GARCH model in the case of daily exchange rate volatility. More importantly, the GARCH forecasting errors show persistent higher order central and non central moments for all empirical time series.

3.1 ARMA-GARCH residuals and Higher Order Cumulants

The concept of “Sufficient statistics“ was introduced by R. A. Fisher in the 1920s. Parametric sufficiency means that the statistics contains just as much information about (some) parameter of the model as the full data. More precisely: the actual data have a certain probability distribution conditional on the data, which in general will also involve the parameter.

By definition, stochastic process Zt is called a Gaussian process if each of its finite dimensional distributions, if the distribution of Zis determined by its mean function μZ(t) = E[Z(t)], and its covariance function Z(s, t) = Cov(Z(s).Z(t)), known as its sufficient statistics.

The aim of this analysis is to shift the tendency of the researchers to assume that the autocovariance function of the stock market squared returns constitutes sufficient statistics for the SM volatility modelling. Thus, in this section the GARCH residuals, obtained in the first step, are whitened subsequently by using the ARMA-HOC estimation method explained above. Ultimately, the comparison is made between: the higher order cumulants of squared returns, the higher order

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cumulants of GARCH residuals and the higher order cumulants of GARCH-HOC residuals. Both third and fourth order cumulants are calculated. For brevity, this analysis is shown only for Swiss stock market index (SMI), for EURO/JPY and GBP/USD exchange rate.

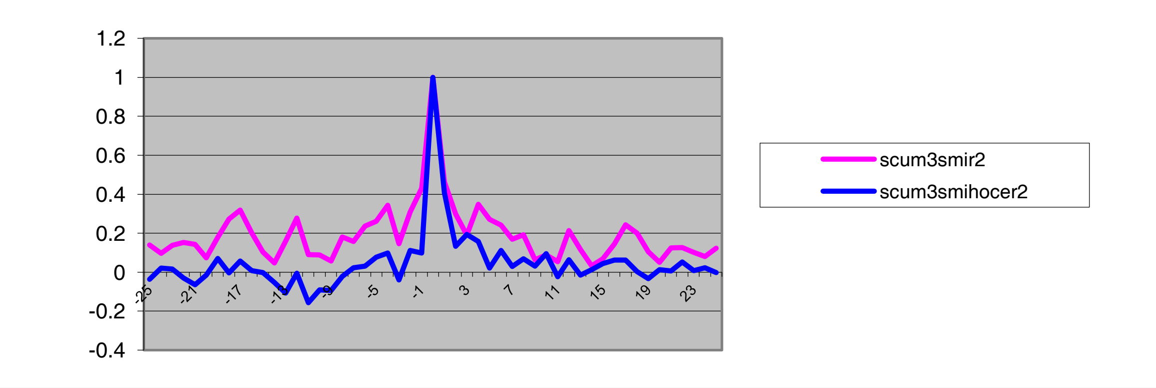


Figure 5 : Third order diagonal cumulants of the GARCH SMI residuals and GRACH-HOC residuals

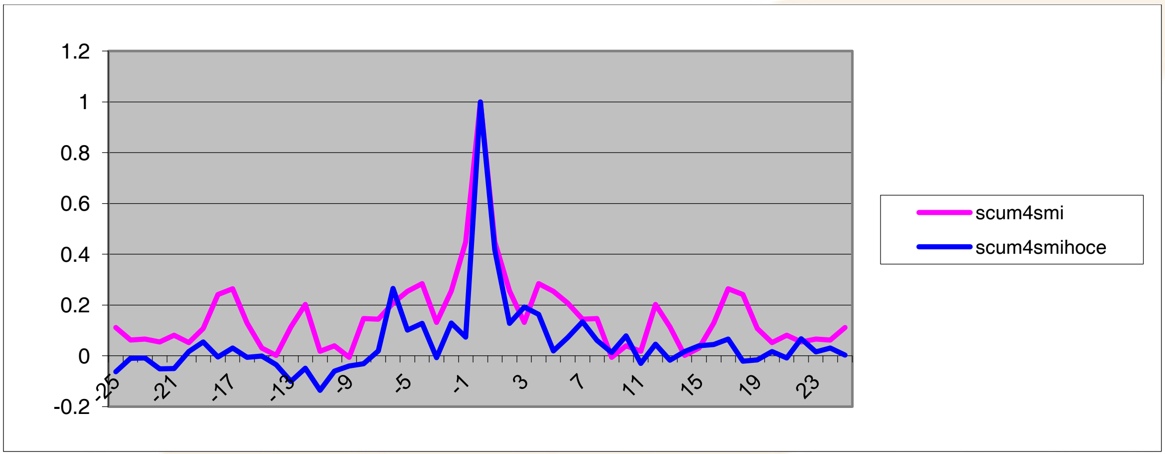


Figure 6 : Forth order diagonal cumulants of the ARMA SMI residuals and ARMA -HOC residuals

It can be seen that residuals obtained by using only the second order cumulants still contain more information about the model , which is minimized when HOC are used .

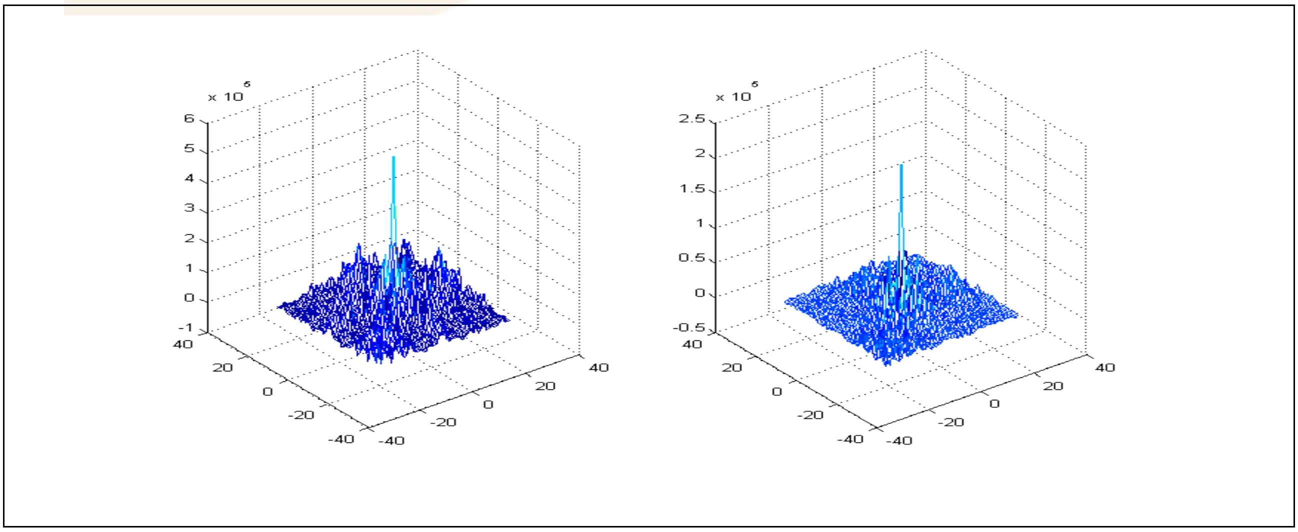


Figure 7 : Third order cumulants of the GARCH SMI residuals and GRACH-HOC residuals (all slices)

 ISBN : 9780974211428

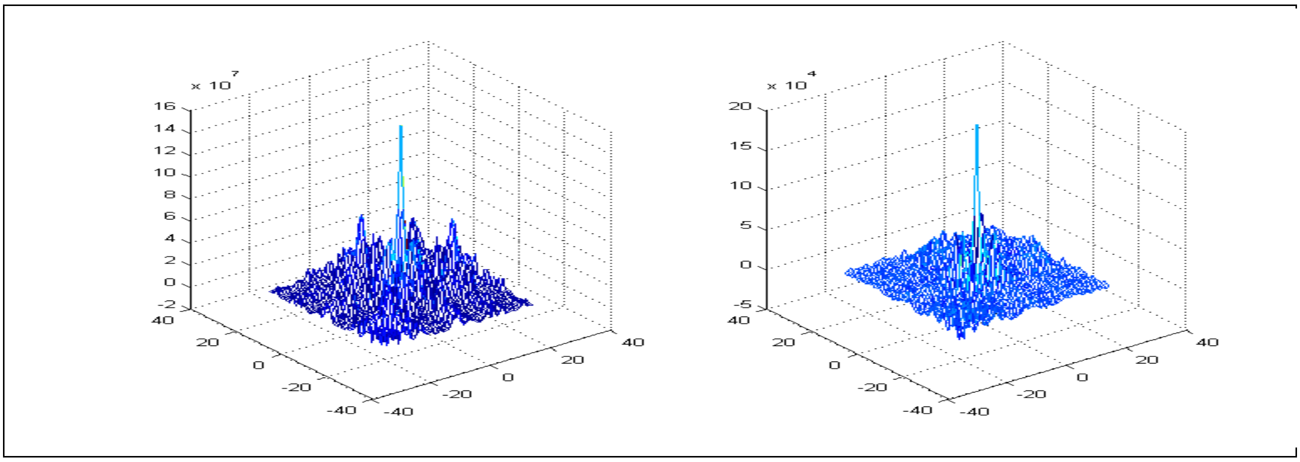
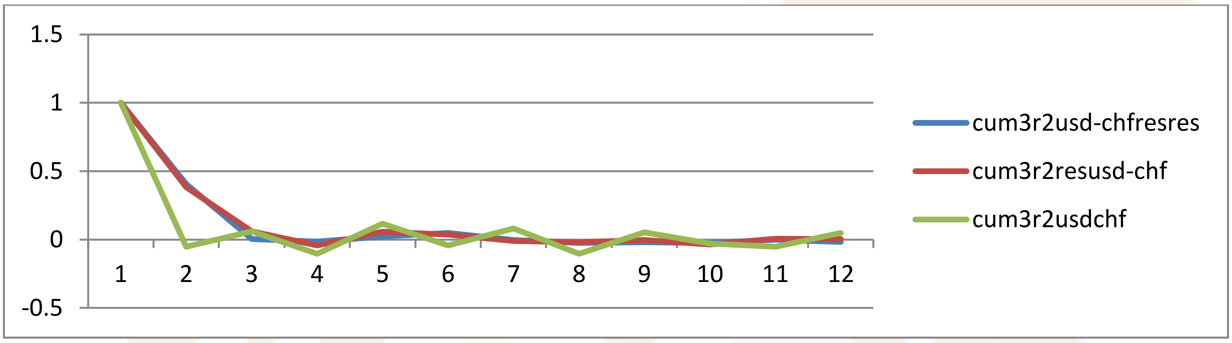
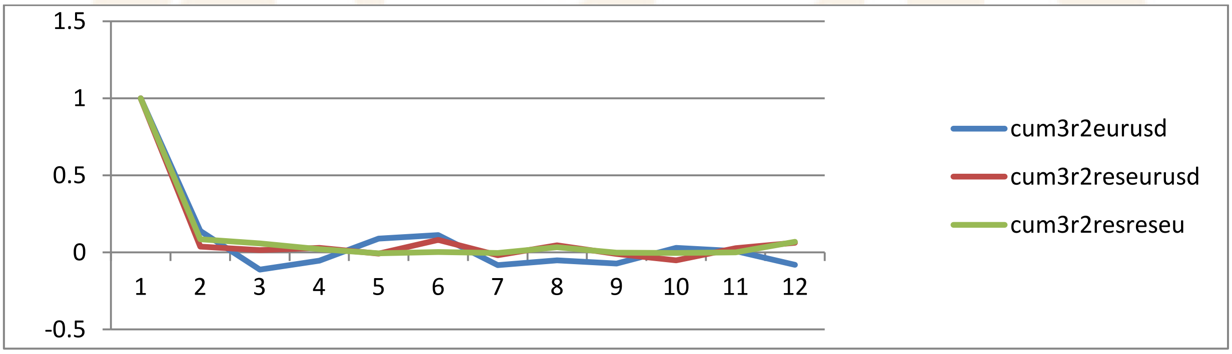
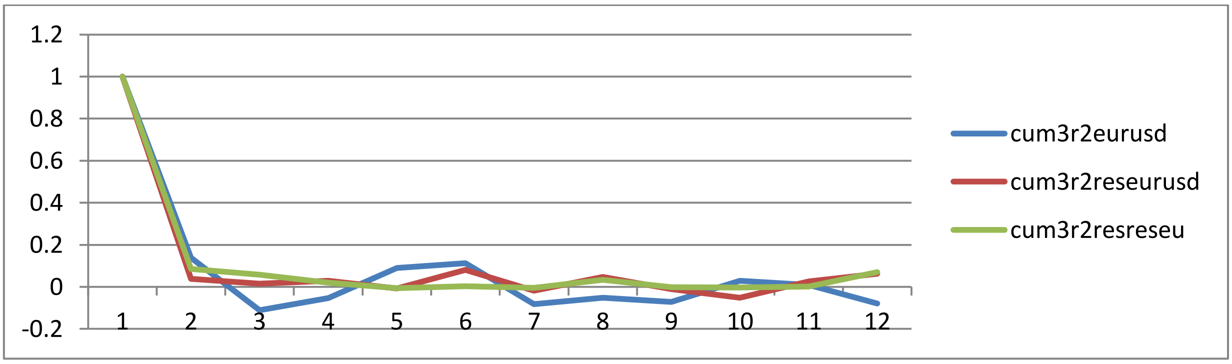


Figure 8 : Forth order cumulants of the ARMA SMI residuals and ARMA -HOC residuals (all slices)







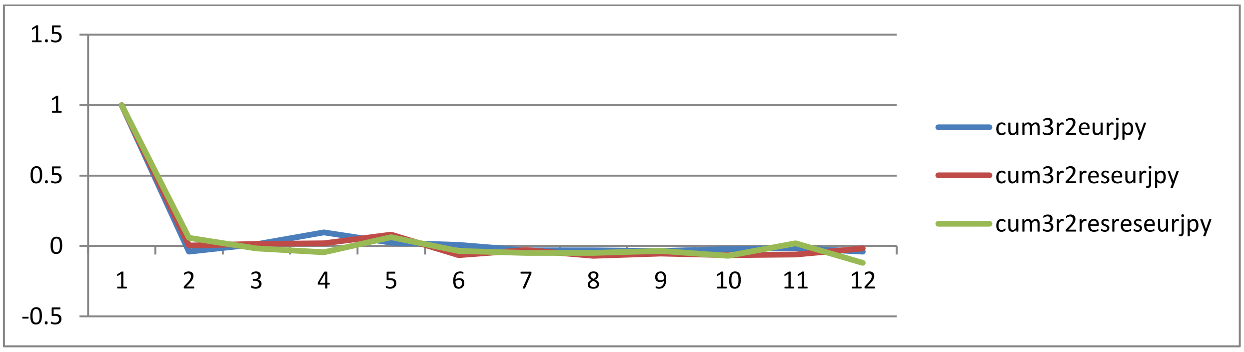


Figure 9. Third order cumulants of FX squared returns, GARCH residuals and ARMA- HOC residuals

These figures demonstrate that the estimation method based on higher order cumulants extracts more information from the sample than regular ML method which is based on the second order cumulants exclusively.

4. Conclusion

It is well known that in the area of financial modeling the GARCH paradigm has been established. Therefore, the large amount of research in the world has been passionately dedicated to the validation of the GARCH models. However, it became difficult to accept the invalidity of the pre-set GARCH idea to use prediction of squared returns as a proxy for volatility forecast and to estimate it by using only the second order moments of squared returns. This article is an attempt to provide evidence that the GARCH paradigm leaves the important problem of defining what is the "sufficient statistics" for financial modeling, unsolved.

It was proven throughout this empirical analysis that neither GARCH-type models nor RV models can capture stylised facts existing in FX squared returns if parameter estimation is based on the second order statistics given the fact that the HOC function of the residuals is not flat as expected. This paper proposes the use of the third and the fourth higher order cumulant functions for the estimation of ARMA parameters. It is empirically demonstrated that digital whitening appears to be more efficient when higher order cumulants are used. Further research needs to address the problem of the stability of the existing algorithms for HOC-ARMA parameter estimation and their optimisation and needs to answer the issue: What is "Sufficient Statistics" for financial modelling?

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